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Neutral stochastic differential equations driven by a fractional Brownian motion with impulsive effects and varying-time delays

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ABSTRACT

In this paper we investigate the existence, uniqueness and asymptotic behaviors of mild solutions to neutral stochastic differential equations with delays and nonlinear impulsive effects, driven by fractional Brownian motion with the Hurst index $H > \frac{1}{2}$ in a Hilbert space. The cases of finite and infinite delays are discussed separately.

1. Introduction

The fractional Brownian motion (fBm) is a family of centered Gaussian processes with continuous sample paths indexed by the Hurst parameter $H \in (0, 1)$. It is a self-similar process with stationary increments and has a long-memory when $H > \frac{1}{2}$. These significant properties make fractional Brownian motion a natural candidate as a model for noise in a wide variety of physical phenomena, such as mathematical finance, communication networks, hydrology and medicine. Therefore, it is important to study stochastic calculus with respect to fBm and related problems (we refer the reader to Mishura (2008) and the references therein for a more complete presentation of this subject).

Recently, stochastic delay differential equations driven by fBm have attracted a lot attentions of works. The first results are established by Ferrante and Rovira (2006). Since then, based on different settings, various forms of equations have been studied. For example, the case of finite-dimensional equations has been studied by Besalú and Rovira (2012); Boufoussi and Hajji (2011); Dung (2012); León and Tindel (2012); Neuenkirch, Nourdin, and Tindel (2008) and the case of equations in a Hilbert space has been considered by Boufoussi and Hajji (2012); Caraballo, Garrido-Atienza, and Taniguchi (2011) and Boufoussi, Hajji, and Lakhel (2012). In most of the works, the delays are finite.

On the other hand, it is known that the impulsive effects exist widely in different areas of real world such as mechanics, electronics, telecommunications, neural networks, finance and economics (see, for instance, Lakshmikantham, Bainov, and Simeonov (1989)). This is due to the fact that the states of many evolutionary processes are often subject to instantaneous perturbations and experience abrupt changes at certain moments of time. The duration of these changes is very short

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and negligible in comparison with the duration of the process considered, and can be thought of as impulses. Hence, it is important to take into account the effect of impulses in the investigation of stochastic delay differential equations driven by fBm. However, to our best knowledge, no work has been reported in the present literature regarding the theory of stochastic differential equations driven by fBm with impulsive effects. The aim of this paper is to study one of such equations.

Our work is inspired by the one of Boufoussi and Hajji (2012) where the following neutral stochastic differential equation driven by fBm with finite delays has been studied

$$\begin{cases} d[x(t)+g(t,x(t-r(t)))] = \big[Ax(t)+f(t,x(t-\rho(t)))\big]dt+\sigma(t)dW^H(t), \quad t\geq 0,\\ x(t)=\phi(t), \quad t\in[-\tau,0]\ (0<\tau<\infty). \end{cases}$$

In this paper, we are interested in the existence, uniqueness and asymptotic behaviors of mild solutions for a neutral stochastic differential equation with finite or infinite delays and impulsive effects of the following form in a Hilbert space

$$\begin{cases} d[x(t)+g(t,x(t-r(t)))] = \big[Ax(t)+f(t,x(t-\rho(t)))\big]dt+\sigma(t)dW^H(t), \quad t\geq 0,\ t\neq t_k,\\ \Delta x(t_k):=x(t_k^+)-x(t_k)=I_k(x(t_k)), \quad k\in\mathbb{N},\\ x(t)=\phi(t), \quad t\in(-\tau,0]\ (0<\tau\leq\infty), \end{cases} \tag{1.1}$$

where A is the infinitesimal generator of an analytic semigroup of bounded linear operators, $(S(t))_{t\geq 0}$, in a Hilbert space X with norm $\|.\|$, W^H is a fractional Brownian motion with $H>\frac{1}{2}$ on a real and separable Hilbert space Y, $r,\rho:[0,\infty)\to[0,\tau)$ are continuous, $\mathbb{N}$ denotes the set of positive integers, the impulsive moments satisfy $0<t_1<t_2<\cdots$, $\lim_{k\to\infty}t_k=\infty$, and $f,g:[0,\infty)\times X\to X$, $\sigma:[0,\infty)\to\mathcal{L}_2^0(Y,X)$, $I_k:X\to X$ are defined later, the initial data $\phi\in C((-\tau,0],X)$ the space of all continuous functions from $(-\tau,0]$ to X and has finite second moments. The space $\mathcal{L}_2^0(Y,X)$ will be defined in Section 2.

It is known that a fBm is neither a semimartingale nor a Markov process. Hence, the traditional tools of Itô stochastic analysis cannot be applied effectively in studying the solution of equations driven by fBm. Because of those reasons, even in the case of equations without impulses, the asymptotic behaviors of solutions have only been investigated by a few authors (see e.g. Boufoussi and Hajji (2012); Caraballo et al. (2011); Duncan, Maslowski, and Pasik-Duncan (2005)). Furthermore, since the appearance of impulses in Eq. (1.1), we need to find the new techniques which are different from that used by Boufoussi and Hajji (2012) to investigate the asymptotic behaviors of solutions of (1.1). The main tool of this paper is the fixed point theory which was proposed by Burton (2006).

The rest of this paper is organized as follows. In Section 2, we briefly present some basic notations and preliminaries. Section 3 is devoted to study the existence, uniqueness and asymptotic behaviors of mild solutions; an example is also provided in this section. The conclusion is given in Section 4.

2. Preliminaries

We first recall the definition of Wiener integrals with respect to an infinite dimensional fractional Brownian motion with Hurst index $H>\frac{1}{2}$.

Let $(\Omega,\mathcal{F},P)$ be a complete probability space and $T>0$ be an arbitrary fixed horizon. An one-dimensional fractional Brownian motion (fBm) with Hurst parameter $H\in(0,1)$ is a centered Gaussian process $\beta^H=\{\beta^H(t), 0\leq t\leq T\}$ with the covariance function $R(t,s)=E[\beta^H(t)\beta^H(s)]$

$$R(t,s)=\frac{1}{2}(|t|^{2H}+|s|^{2H}-|t-s|^{2H}).$$

It is known that $\beta^H(t)$ with $H>\frac{1}{2}$ admits the following Volterra representation

$$\beta^H(t)=\int_0^t K(t,s)d\beta(s), \tag{2.1}$$

where β is a standard Brownian motion and the Volterra kernel $K(t,s)$ is given by

$$K(t,s)=c_H\int_s^t (u-s)^{H-\frac{3}{2}}\left(\frac{u}{s}\right)^{H-\frac{1}{2}}du, \quad t\geq s.$$

For the deterministic function $\varphi\in L^2([0,T])$, the fractional Wiener integral of φ with respect to β^H is defined by

$$\int_0^T\varphi(s)d\beta^H(s)=\int_0^T K_H^*\varphi(s)d\beta(s),$$

where $K_H^*\varphi(s)=\int_s^T\varphi(r)\frac{\partial K}{\partial r}(r,s)dr$.

Let X and Y be two real, separable Hilbert spaces and let $\mathcal{L}(Y,X)$ be the space of bounded linear operators from Y to X. For the sake of convenience, we shall use the same notation to denote the norms in X, Y and $\mathcal{L}(Y,X)$. Let $\{e_n, n=1,2,\ldots\}$ be a complete orthonormal basis in Y and $Q\in\mathcal{L}(Y,X)$ be an operator defined by $Qe_n=\lambda_ne_n$ with finite trace $trQ=$