Journal of the Korean Statistical Society 43 (2014) 415–424

Contents lists available at ScienceDirect

Journal of the Korean Statistical Society

journal homepage: www.elsevier.com/locate/jkss

Analyses of the Markov modulated fluid flow with one-sided ph-type jumps using coupled queues and the completed graphs

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ARTICLE INFO

Article history:
Received 16 December 2013
Accepted 26 December 2013
Available online 21 January 2014

AMS 2000 subject classifications:
60K25
90B05

Keywords:
Markov modulated fluid flow model with jumps
Riccati equation
Duality
ph-type distribution
The completed graph

ABSTRACT

In this paper, we analyze Markov modulated fluid flow processes with one-sided ph-type jumps using the completed graph and also through the limits of coupled queueing processes to be constructed. For the models, we derive various results on time-dependent distributions and distributions of first passage times, and present the Riccati equations that transform matrices of the first return times to 0 satisfy. The Riccati equations enable us to compute the transform matrices using Newton's method which is known very fast and stable. Finally, we present some duality results between the model with ph-type downward jumps and the model with ph-type upward jumps. This paper contains extended results of Ahn (2009) and probabilistic interpretations given by the completed graphs.

1. Introduction

In this paper, we consider Markov modulated fluid flow (MMFF) models with one-sided ph-type jumps under general conditions. The restriction on the jump distributions is not so restrictive due to the denseness property of the class of ph-type distributions. For details on the ph-type distribution, we refer to Latouche and Ramaswami (1999).

The MMFF processes with jumps have their various applications especially in risk modeling and insurance ruin theories (Ahn, Badescu, & Ramaswami, 2007; Ramaswami, 2005). There have been several papers, e.g. Miyazawa and Takada (2002), Sengupta (1989) and Tzenova, Adan, and Kullarni (2005), which deal with stationary analyses of the considered models. Sengupta (1989) derives the steady state distribution of a Markov fluid model with downward jumps using matrix integral equations, which is generalized by Miyazawa and Takada (2002). On the other hand, Tzenova et al. (2005) consider a Markov fluid model with upward jumps and derive its steady state distribution using differential equations and eigenvalue theories. Recently Ahn (2009) presents several results on time-dependent distributions and the first passage times of the MMFF processes with one-sided ph-type jumps by constructing a sequence of approximating coupled queues as in Ahn and Ramaswami (2004). But, their models have some restrictions such that the fluid level in the model with upward jumps cannot increase linearly and the fluid level in the model with downward jumps cannot decrease linearly.

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http://dx.doi.org/10.1016/j.jkss.2013.12.004

In this paper, we relax the restrictions of the model in Ahn (2009) so that the fluid level continuously increases, decreases and also can stay level unchanged in addition to one-sided ph-type jumps. Then, for the model with downward jumps, we derive Laplace–Stieltjes transforms (LST) of the first passage times and Laplace transforms (LT) of the time-dependent distributions. We also derive Laplace–Stieltjes transforms of the first passage times for the model with upward jumps and present duality results between the models respectively with upward jumps and downward jumps.

For the analyses, we construct sequences of coupled queues in relation to the models and use their limits as in Ahn (2009) and Ahn and Ramaswami (2004), and also adopt the completed graph to be defined for the first time in the present paper. Further, we show that all the quantities of interest can be represented by the transform matrices of the first return times of the completed graphs, which is one of the contributions of the present paper. It is also an important contribution that we provide Riccati equations whose minimal non-negative solutions are these transform matrices. The solutions can be obtained using the algorithm so called Newton's method, which is introduced in Guo (2001) and is known simple, fast and stable.

The remaining parts of this paper is organized as follows. In Section 2, We give model descriptions and introduce notations to be used throughout the paper. In the same section, we also introduce the coupled queues and the completed graphs in relation to the MMFF processes with one-sided ph-type jumps. In Sections 3 and 4, we analyze the MMFF process with downward ph-type jumps and the MMFF process with upward ph-type jumps respectively. Finally in Section 5, we present duality results existing between the MMFF process with respectively downward and upward ph-type jumps.

2. Model descriptions and notations

2.1. Model descriptions and the completed graphs

Before we define the MMFF processes with on-sided ph-type jumps, we first introduce *free MMFF process without jumps*. A free MMFF process without jumps with state space S, infinitesimal generator Q and rate vector $\mathbf{r} = (r_i, i \in S)$, denoted by $(\mathcal{M}^*, \mathcal{J}^*) = \{(M^*(t), J^*(t)), t \geq 0\}$, is a two-dimensional continuous-time Markov process, where the phase process $\mathcal{J}^*$ is an irreducible continuous-time Markov process with the state space S and the infinitesimal generator Q, and the level process $\mathcal{M}^*$ is defined as

$$M^*(t) = \int_0^t r_{J^*(y)} dy. \tag{1}$$

We assume that the state space $S = S_u \cup S_d \cup S_{j^-} \cup S_0$ when we analyze the MMFF process with downward ph-type jumps and in this case we use a notation $(\mathcal{M}^*_-, \mathcal{J}^*) = \{(M^*_-(t), J^*(t)), t \geq 0\}$ instead of $(\mathcal{M}^*, \mathcal{J}^*)$. We also assume that the state space $S = S_u \cup S_{j^+} \cup S_d \cup S_0$ when we analyze the MMFF process with upward ph-type jumps and in this case we use a notation $(\mathcal{M}^*_+, \mathcal{J}^*) = \{(M^*_+(t), J^*(t)), t \geq 0\}$ instead of $(\mathcal{M}^*, \mathcal{J}^*)$. In relation to the rate vector $\mathbf{r}$, it is assumed that $r_i > 0, i \in S_u, r_i < 0, i \in S_d, r_i = 1, i \in S_{j^+}, r_i = -1, i \in S_{j^-}$ and $r_i = 0, i \in S_0$.

It is useful for later analyses to represent the infinitesimal generator Q in the following partitioned form in conformity with the partition of the state space S as

$$Q = (Q_{l,m}, l, m = u, d, j^-, 0) \quad \text{when } S = S_u \cup S_d \cup S_{j^-} \cup S_0 \quad \text{or}$$
$$Q = (Q_{l,m}, l, m = u, j^+, d, 0) \quad \text{when } S = S_u \cup S_{j^+} \cup S_d \cup S_0.$$

In relation to the free MMFF processes without jumps, we define restricted processes $(\mathcal{M}_-, \mathcal{J})$ and $(\mathcal{M}_+, \mathcal{J})$ as the processes which are obtained through observing the processes $(\mathcal{M}^*_-, \mathcal{J}^*)$ and $(\mathcal{M}^*_+, \mathcal{J}^*)$ only when $\mathcal{J}^*$ stays in $S_u \cup S_d \cup S_0$. To state this more precisely, we let a time-shift function $\eta(t) = \inf\{v > 0 : \int_0^v \mathbf{1}_{S_u \cup S_d \cup S_0}(J^*(y))dy > t\}$ and define $\mathcal{M}_-$, $\mathcal{M}_+$ and $\mathcal{J}$ such as $M_-(t) = M^*_-(\eta(t))$, $M_+(t) = M^*_+(\eta(t))$ and $J(t) = J^*(\eta(t))$. Then we can represent the processes $\mathcal{M}_-$ and $\mathcal{M}_+$ as

$$M_-(t) = \int_0^t r_{J(y)} dy - \sum_{i=1}^{N_-(t)} X_i \quad \text{and} \quad M_+(t) = \int_0^t r_{J(y)} dy + \sum_{i=1}^{N_+(t)} Y_i, \tag{2}$$

where $N_-(t)$ and $N_+(t)$ are the number of transitions of $\mathcal{J}^*$ in $(0, \eta(t)]$ from $S_u \cup S_d \cup S_0$ to S_{j^-} or S_{j^+} respectively, and the random variables X_i and Y_i denote the length of ith dwelling time of $\mathcal{J}^*$ in S_{j^-} and S_{j^+} respectively when such transitions happen. We call the processes $(\mathcal{M}_-, \mathcal{J})$ and $(\mathcal{M}_+, \mathcal{J})$ a *free MMFF process with respectively downward and upward ph-type jumps* with state space S, infinitesimal generator Q and rate vector $\mathbf{r}$.

Now, we are ready to define the MMFF processes with one-sided ph-type jumps of interest in the present paper which are represented using the reflection map, where the reflection map $\mathcal{R}$ is a mapping defined such as for given a real-valued process $\mathcal{A}$, $\mathcal{R} \circ \mathcal{A}(t) = A(t) - \min\{0, \min_{0 \leq u \leq t} A(u)\}$. We first define level processes $\mathcal{F}_-$ and $\mathcal{F}_+$ such that

$$F_-(t) = \mathcal{R} \circ \mathcal{M}_-(t) \quad \text{and} \quad F_+(t) = \mathcal{R} \circ \mathcal{M}_+(t), \quad t \geq 0.$$

Then we call the processes $(\mathcal{F}_-, \mathcal{J})$ and $(\mathcal{F}_+, \mathcal{J})$ a *MMFF process with respectively downward and upward ph-type jumps* with state space S, infinitesimal generator Q and rate vector $\mathbf{r}$.