



Quantile based reliability aspects of partial moments

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ABSTRACT

Partial moments are extensively used in literature for modeling and analysis of lifetime data. In this paper, we study properties of partial moments using quantile functions. The quantile based measure determines the underlying distribution uniquely. We then characterize certain lifetime quantile function models. The proposed measure provides alternate definitions for ageing criteria. Finally, we explore the utility of the measure to compare the characteristics of two lifetime distributions.

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1. Introduction

Let X be a random variable with distribution function $F(x)$ and finite moment of order r . Then the r th upper partial moment about x is defined as

$$\alpha_r(x) = E(X - x)_+^r = \int_x^\infty (t - x)^r dF(t) \quad (1.1)$$

where $(X - x)_+ = \max[(X - x), 0]$. The quantity $(X - x)_+$ is interpreted as the residual age in the context of lifelength studies (Lin, 2003) and the first two moments and variance of $(X - x)_+$ are used in actuarial studies for the analysis of risks (Denuit, 2002). In the assessment of income, x can be taken as the tax exemption level so that $(X - x)_+$ becomes the taxable income.

Gupta and Gupta (1983) have discussed general properties of partial moments. They proved that (1.1) determines the underlying distribution uniquely for any positive real r . Also when r is a positive integer there exists a recurrence relation between two consecutive partial moments. Abraham, Nair, and Sankaran (2007), Chong (1977) and Lin (2003) have characterized the exponential, beta and Pareto II distributions by relationships among various moments. The survival function $\bar{F}(x)$ of X can be written in terms of $\alpha_r(x)$ as (Navarro, Franco, & Ruiz, 1998; Sunoj, 2004)

$$\bar{F}(x) = \frac{(-1)^r}{r!} \frac{d^r \alpha_r(x)}{dx^r}.$$

Gupta (2007) and Sunoj (2004) obtained partial moments and their properties in respect of length biased and equilibrium distributions.

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All these theoretical results and applications thereof use the definition (1.1) based on distribution function. A probability distribution can be specified either in terms of the distribution function or by the quantile function.

$$Q(u) = \inf_x \{x : F(x) \geq u\}, \quad 0 \leq u \leq 1$$

and recently there has been substantial interest in modeling statistical data using quantile functions. Many of the quantile functions used in statistical theory and applications like various forms of lambda distributions (Freimer, Mudholkar, Kollia, & Lin, 1998; Gilchrist, 2000; Ramberg & Schmeiser, 1974; van Staden & Loots, 2009), the power-Pareto distribution (Gilchrist, 2000; Hankin & Lee, 2006), Govindarajulu distribution (Nair, Sankaran, & Vineshkumar, 2012) etc. do not have tractable distribution functions.

For example in the case of the Govindarajulu distribution specified by

$$Q(u) = \sigma[(\beta + 1)u^\beta - \beta u^{\beta+1}], \quad 0 \leq u \leq 1, \quad \sigma, \beta > 0$$

cannot be inverted from $Q(u) = x$ to $u = F(x)$ analytically. In practice, for purposes of analysis the procedure is to solve for u from the equation

$$x = \sigma((\beta + 1)u^\beta - \beta u^{\beta+1})$$

using numerical techniques, corresponding to chosen values of x . Such a collection of values of $F(x)$ is insufficient to determine the characteristics of the distribution exactly. The only alternative to resolve the problem is to find quantile-based equivalents of the definitions of the characteristics and then use them for theoretical analysis. An illustration of this approach that led to new methodology, analysis and models in the context of reliability analysis can be seen in Nair and Sankaran (2009), Nair, Sankaran, and Vineshkumar (2011) and Nair and Vineshkumar (2010). These works also indicate that even when the quantile functions are invertible, the approach provides alternative methodologies that have desirable properties and are easier for applications.

Thus a formulation of the definition and properties of partial moments in terms of quantile functions is essential to study them in the context of the quantile function model. Such a discussion has several advantages. Analytical properties of the partial moments obtained in this approach can be used as an alternative tool in modeling statistical data. Sometimes the quantile based approach is better in terms of tractability. New models and characterizations that are unresolvable in the distribution functions approach can be resolved with the aid of the quantile approach. For example, the sum of two quantile functions is again a quantile function. Hence starting with a partial mean of a quantile function, one can add another partial mean to generate a new partial mean and the corresponding new quantile function. This imparts considerable flexibility in modeling problems. See Sections 2 and 3 for a further elucidation of this and other aspects of the advantages of the quantile approach. In view of these, the objective of the present work is to initiate a discussion of quantile based partial moments in the context of reliability analysis.

The text is organized as follows. In Section 2, we present the definition of partial moments in terms of the quantile function. Various properties of the partial moments and their relationships with other basic reliability concepts are discussed. The proposed definition is used, in Section 3, to characterize certain lifetime quantile function models. The potential of quantile-based definition to characterize various notions of ageing is studied in Section 4. In Section 5 we demonstrate the utility of some of the results by applying them to a real data. Finally, Section 6 provides the utility of the new definition to compare characteristics of two life distributions.

2. Basic results

Let X be a non-negative random variable with absolutely continuous distribution function $F(x)$ and probability density function $f(x)$. When $F(x)$ is strictly increasing, the quantile function $Q(u)$ is obtained from the solution of $F(x) = u$, as $x = Q(u)$. The mean of the distribution, when $Q(0) = 0$ is given by

$$\mu = \int_0^1 Q(p) dp \tag{2.1}$$

$$= \int_0^1 (1 - p)q(p) dp \tag{2.2}$$

where $q(u) = \frac{dQ(u)}{du}$ is the quantile density function.

Since $F(x)$ is strictly increasing, $f(x) > 0$ so that the quantile density function $q(u)$ exists by virtue of the relation

$$q(u)f(Q(u)) = 1. \tag{2.3}$$

The quantile version of the partial moment is obtained by setting $F(x) = u$ in (1.1), which gives

$$P_r(u) = \alpha_r(Q(u)) = \int_u^1 (Q(p) - Q(u))^r dp. \tag{2.4}$$

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