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Anticipated BSDEs driven by time-changed Lévy noises☆

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ABSTRACT

In this paper, we discuss a class of anticipated backward stochastic differential equations (anticipated BSDEs, in short) driven by time-changed Lévy noises. We establish the existence and uniqueness of the solution. Moreover, we establish the duality relation between stochastic differential delay equations (SDDEs, in short) and anticipated BSDEs driven by time-changed Lévy noises.

1. Introduction

Since Pardoux and Peng (1990) introduced the theory of nonlinear backward stochastic differential equation (BSDE, in short), many interesting kinds of BSDEs have been established for their applications in many fields. Especially, a lot of works have been devoted to the study of BSDEs driven by different noises as well as their applications. One can see Cohen and Hu (2012), Cohen and Szpruch (2012) and the references therein. Moreover, Peng and Yang (2009) introduced a new kind of BSDEs, called anticipated as BSDEs with the following form:

$$\begin{cases} dY_t = -f(t, Y_t, Z_t, Y_{t+u(t)}, Z_{t+v(t)})dt + Z_t dW_t, & t \in [0, T], \\ Y_t = \xi_t, & t \in [T, T+K], \\ Z_t = \eta_t, & t \in [T, T+K], \end{cases} \tag{1}$$

where $u(\cdot) : [0, T] \to \mathbb{R}^+ \backslash \{0\}$ and $v(\cdot) : [0, T] \to \mathbb{R}^+ \backslash \{0\}$ are continuous functions satisfying that

(i) there exists a constant $K \geq 0$ such that for each $t \in [0, T]$,

$$t + u(t) \leq T + K, \qquad t + v(t) \leq T + K;$$

(ii) there exists a constant $L \geq 0$ such that for each $t \in [0, T]$ and each nonnegative integrable function $g(\cdot)$,

$$\int_t^T g(s + u(s))ds \leq L \int_t^{T+K} g(s)ds, \qquad \int_t^T g(s + v(s))ds \leq L \int_t^{T+K} g(s)ds.$$

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In Eq. (1), the generator f contains not only the values of solutions of present but also the future. In Peng and Yang (2009), the authors proved the existence and uniqueness of the adapted solution and gave a comparison theorem for the solutions of anticipated BSDEs. Furthermore, they gave a duality relation between stochastic differential delay equations (SDDEs, in short) and anticipated BSDEs, which is a useful tool in the analysis of stochastic optimal control problems (see e.g. Chen & Wu, 2010; Yu, 2012). Furthermore, Xu (2011) established a necessary and sufficient condition for the comparison theorem of multidimensional anticipated BSDEs. Based on the work of Cohen and Elliott (2008), Lu and Ren (2013) proved the existence and uniqueness of the solution for anticipated BSDEs related to finite state, continuous time Markov chains.

Very recently, Di Nunno and Sjursen (2014) introduced a class of BSDEs driven by time-changes Lévy process, which is constructed by a conditional Brownian motion and a doubly stochastic Poisson random field of the form

$$Y_t = \xi + \int_t^T g_s(\lambda_s, Y_s, \phi_s)ds - \int_t^T \int_{\mathbb{R}} \phi_s(x)\mu(ds, dx), \quad t \in [0, T], \tag{2}$$

where μ is the mixture of a conditional Brownian measure B on $[0, T]\times\{0\}$ and a centered doubly stochastic Poisson measure $\tilde{H}$ on $[0, T] \times \mathbb{R}_0$ with

$$\mu(\triangle) := B(\triangle \cap [0, T] \times \{0\}) + \tilde{H}(\triangle \cap [0, T] \times \mathbb{R}_0).$$

The authors proved the existence and uniqueness of the solution and established the sufficient conditions for the maximum principle for a general optimal of a system by a time-changed Lévy noise.

Motivated by the above works, the present paper aims to discuss a class of anticipated BSDEs driven by time-changed Lévy noises of the form

$$\begin{cases} dY_t = -f\left(t, \lambda_t, Y_t, Z_t, Y_{t+u(t)}, Z_{t+v(t)}\right) dt + Z_t(x)\mu(dt, dx), & t \in [0, T], \\ Y_t = \xi_t, & t \in [T, T+K], \\ Z_t = \eta_t, & t \in [T, T+K]. \end{cases} \tag{3}$$

As the first step, we aim to give the existence and uniqueness of the solution for the above equations. Moreover, the duality relation between SDDEs and anticipated BSDEs will also be established. In a future study, we will obtain the maximal principle for the optimal control of the stochastic differential equations with time-changed Lévy noises by means of the results established in this paper.

The present paper is organized as follows. In Section 2, we introduce some preliminaries and notations. Section 3 is devoted to prove the existence and uniqueness of the solution. In Section 4, we establish the duality relation between SDDEs and anticipated BSDEs driven by time-changed Lévy noises.

2. Preliminaries and notations

In this section, we recall some concepts and auxiliary results to establish our desired results. For more details, one can see Di Nunno and Sjursen (2014) and the references therein.

Let $T > 0$ be fixed, $(\Omega, \mathcal{F}, P)$ be a complete probability space and $X := [0, T] \times \mathbb{R}$. In the sequel, let $X = ([0, T] \cup \{0\}) \cup ([0, T] \times \mathbb{R}_0)$, where $\mathbb{R}_0 = \mathbb{R} \setminus \{0\}$.

Let $\lambda = (\lambda^B, \lambda^H)$ be a two dimensional stochastic process that each component λ^i, $i = B, H$ satisfies that

(a1) for every $t \in [0, T]$, $\lambda_t^i \geq 0$ $a.s.$;
(a2) for $\forall \epsilon > 0$, and $t \in [0, T]$, $\lim_{h\to 0} P(|\lambda_{t+h}^i - \lambda_h^i| \geq \epsilon) = 0$;
(a3) $\mathrm{E}[\int_0^T \lambda_t^i dt] < +\infty$.

Define the random measures Λ on X by

$$\Lambda(\Delta) := \int_0^T \mathbf{1}_{\{(t,0)\in\Delta\}}(t)\lambda_t^B dt + \int_0^T \int_{\mathbb{R}_0} \mathbf{1}_\Delta(t, x)q(dx)\lambda_t^H dt$$
$$:= \Lambda^B(\Delta \cap [0, T] \times \{0\}) + \Lambda^H(\Delta \cap [0, T] \times \{\mathbb{R}_0\})$$

as the mixture of measure on disjoint sets, $\Delta \subseteq X$. Here, we denote $\mathcal{F}^\Lambda$ as the σ-algebra generated by the values of Λ, and q is a deterministic, σ-finite measure on the Borel sets of $\mathbb{R}_0$ satisfying that

$$\int_{\mathbb{R}_0} x^2 q(dx) < +\infty.$$

Now, we introduce the noises which drive the anticipated BSDE (3).

Definition 1. B is a signed random measure on the Borel sets of $[0, T] \times \{0\}$ satisfying that

(b1) $P\big(B(\Delta) \leq x|\mathcal{F}^\Lambda\big) = P\left(B(\Delta) \leq x|\Lambda^B(\Delta)\right) = \Phi\left(\frac{x}{\sqrt{\Lambda^B(\Delta)}}\right)$, $x \in \mathbb{R}$, $\Delta \subseteq [0, T] \times \{0\}$, where $\Phi(\cdot)$ is the cumulative probability distribution function of a standard normal random variable.
(b2) $B(\Delta_1)$ and $B(\Delta_2)$ are conditionally independent given $\mathcal{F}^\Lambda$ whenever Δ_1 and Δ_2 are disjoint sets.