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Option pricing for a stochastic volatility Lévy model with stochastic interest rates

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ABSTRACT

An alternative option pricing model under a forward measure is proposed, in which asset prices follow a stochastic volatility Lévy model with stochastic interest rate. The stochastic interest rate is driven by the Hull–White process. By using an approximate method, we find a formulation for the European option in term of the characteristic function of the tail probabilities.

1. Introduction

Let (Ω, F, P) be a probability space. A stochastic process L_t is a Lévy process if it has independent and stationary increments and has a stochastically continuous sample path, i.e. for any $\varepsilon > 0$, $\lim_{h\downarrow 0} P(|L_{t+h} - L_t| > \varepsilon) \to 0$. The simplest possible Lévy processes are the standard Brownian motion W_t, Poisson process N_t, and compound Poisson process $\sum_{i=1}^{N_t} Y_i$ where Y_i are i.i.d. random variables. Of course, we can build a new Lévy process from known ones by using the technique of linear transformation. For example, the jump diffusion process $\mu t + \sigma W_t + \sum_{i=1}^{N_t} Y_i$, where μ, σ are constants, is a Lévy process which comes from a linear transformation of two independent Lévy processes, i.e. a Brownian motion with drift and a compound Poisson process.

Assume that a risk-neutral probability measure Q exists and all processes in Sections 1 and 2 will be considered under this risk-neutral measure.

In the Black–Scholes model, the price of a risky asset S_t under a risk-neutral measure Q and with non dividend payment follows

$$S_t = S_0 \exp(\tilde{L}_t) = S_0 \exp\left(rt + \left(\sigma W_t - \frac{1}{2}\sigma^2 t\right)\right), \tag{1.1}$$

where $r \in \Re$ is a risk-free interest rate, $\sigma \in \Re$ is a volatility coefficient of the stock price.

Instead of modeling the log returns $\tilde{L}_t = rt + \left(\sigma W_t - \frac{1}{2}\sigma^2 t\right)$ with a normal distribution, we now replace it with a more sophisticated process L_t which is a Lévy process of the form

$$L_t = rt + \left(\sigma W_t - \frac{1}{2}\sigma^2 t\right) + J_t, \tag{1.2}$$

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where J_t denotes a pure Lévy jump component, (i.e. a Lévy process with no Brownian motion part). We assume that the processes W_t and J_t are independent.

To incorporate the volatility effect to the model Eq. (1.2), we follow the technique of Carr and Wu (2004) by subordinating a standard Brownian motion component $\sigma W_t - \frac{1}{2}\sigma^2 t$ and a pure jump Lévy process J_t by the time integral of a mean reverting Cox–Ingersoll–Ross (CIR) process

$$T_t = \int_0^t \upsilon_s ds,$$

where υ_t follows the CIR process

$$d\upsilon_t = \gamma(1 - \upsilon_t)dt + \sigma_\upsilon \sqrt{\upsilon_t} dW_t^\upsilon. \tag{1.3}$$

Here W_t^υ is a standard Brownian motion which corresponds to the process υ_t. The constant $\gamma \in \mathbb{R}$ is the rate at which the process υ_t reverts toward its long term mean and $\sigma_\upsilon > 0$ is the volatility coefficient of the process υ_t.

Hence, the model (1.2) has been changed to

$$L_t = rt + \left(\sigma W_{T_t} - \frac{1}{2}\sigma^2 T_t\right) + J_{T_t} \tag{1.4}$$

and this new process is called a stochastic volatility Lévy process. One can interpret T_t as the stochastic clock process with activity rate process υ_t. By replacing $\tilde{L}_t$ in (1.1) with L_t, we obtain a model of an underlying asset under the risk-neutral measure Q with stochastic volatility as follows:

$$S_t = S_0 \exp(L_t) = S_0 \exp\left(rt + \left(\sigma W_{T_t} - \frac{1}{2}\sigma^2 T_t\right) + J_{T_t}\right). \tag{1.5}$$

In this paper, we shall consider the problem of finding a formula for European call options based on the underlying asset model (1.5) for which the constant interest rates r is replaced by the stochastic interest rates r_t, i.e. the model under our consideration is given by

$$S_t = S_0 \exp\left(r_t t + \left(\sigma W_{T_t} - \frac{1}{2}\sigma^2 T_t\right) + J_{T_t}\right). \tag{1.6}$$

Here, we assume that r_t follows the Hull–White process

$$dr_t = (\alpha(t) - \beta r_t)dt + \sigma_r dW_t^r, \tag{1.7}$$

W_t^r is a standard Brownian motion with respect to the process r_t, and $dW_t^r dW_t^{=}0$. The constant $\beta \in \mathbb{R}$ is the rate at which the interest rate reverts toward its long term mean, $\sigma_r > 0$ is the volatility coefficient of the interest rate process (1.7), $\alpha(t)$ is a deterministic function, and is well defined in a time interval $[0, T]$. We also assume that the interest rate process r_t and the activity rate process υ_t are independent.

The problem of option pricing under stochastic interest rates has been investigated for a long time. Kim (2001) constructed the option pricing formula based on Black–Scholes model under several stochastic interest rate processes, i.e., Vasicek, CIR, Ho–Lee type. He found that by incorporating stochastic interest rates into the Black–Scholes model, for a short maturity option, does not contribute to improvement in the performance of the original Black–Scholes' pricing formula. Brigo and Mercuiro (2001, p. 883) mention that the stochastic feature of interest rates has a stronger impact on the option price when pricing for a long maturity option. Carr and Wu (2004) continue this study by giving the option pricing formula based on a time-changed Levy process model. But they still use constant interest rates in the model.

In this paper, we give an analysis on the option pricing model based on a time-changed Levy process with stochastic interest rates.

The rest of the paper is organized as follows. The dynamics under the forward measure is described in Section 2. The option pricing formula is given in Section 3. Finally, the close form solution for a European call option in terms of the characteristic function is given in Section 4.

2. The dynamics under the forward measure

We begin by giving a brief review of the definition of a correlated Brownian motion and some of its properties (see Brummelhuis (2009, p. 70)). Recalling that a *standard Brownian motion in* R^n is a stochastic process $(\vec{Z}_t)_{t\geq 0}$ whose value at time t is simply a vector of n independent Brownian motions at t:

$$\vec{Z}_t = (Z_{1,t}, \ldots, Z_{n,t}).$$

We use Z instead of W, since we would like to reserve the latter for the more general case of correlated Brownian motion, which will be defined as follows: