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Bayesian reliability when system and subsystem failure data are obtained in the same time period

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ABSTRACT

Previously, Bayesian anomaly was reported for estimating reliability when subsystem failure data and system failure data were obtained from the same time period. As a result, a practical method for mitigating Bayesian anomaly was developed. In the first part of this paper, however, we show that the Bayesian anomaly can be avoided as long as the same failure information is incorporated in the model. In the second part of this paper, we consider a problem of estimating the Bayesian reliability when the failure count data on subsystems and systems are obtained from the same time period. We show that Bayesian anomaly does not exist when using the multinomial distribution with the Dirichlet prior distribution. A numerical example is given to compare the proposed method with the previous methods.

1. Introduction

Reliability is defined as a system's ability to perform a required function under given operating conditions for a fixed period of time. Typically, reliability is measured according to failure rate, survival probability or success probability. Either the classical frequency-based method or the Bayesian method have been used to obtain an accurate estimate of a reliability measure. The Bayesian reliability estimate is preferred to the classical frequency-based estimate when failure data is limited or expert opinion is available (Hamada, Wilson, Reese, & Martz, 2008).

In this paper, we consider a Bayesian reliability estimation method when both system failure data and subsystem failure data are available. Extensive research must be conducted if system failure data and subsystem failure data originate from different time periods. Two distinct approaches are presented for proceeding with data obtained from different time periods. First, the posterior system reliability is obtained using system failure data, from which the prior subsystem reliability is derived. The posterior subsystem reliability is subsequently developed using subsystem failure data (Mastran, 1976). Second, the posterior subsystem reliability is obtained using subsystem failure data and then the posterior system reliability is obtained using system failure data (Martz, Waller, & Fickas, 1988).

However, research is limited for the case in which both subsystem failure data and system failure data are obtained from the same time period. One difficulty in this case is that system reliability obtained at the subsystem level is different from system reliability obtained at the system level. This difference is called the Bayesian anomaly (Philipson, 1995, 2008) or the Bayesian aggregation error (Bier, 1994; Mosleh & Bier, 1992). The Bayesian anomaly limits the meaningfulness of Bayesian analysis (Philipson, 1995) and a restructuring of the Bayesian procedure is required for solving the failure of Bayesian system reliability inference based on subsystem failure data and system failure data (Philipson, 1996). Recently, Philipson (2008) has proposed a method for mitigating the Bayesian anomaly in which the system unreliability estimate obtained from

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system failure data was allocated to the subsystems proportionally to the subsystem unreliability estimate obtained from the subsystem failure data.

This paper consists of two parts. The first part of this paper investigates the source of the Bayesian anomaly, and shows that the Bayesian anomaly is not caused by the Bayesian procedure but caused by different failure information incorporated in the model. Therefore, the Bayesian anomaly can occur even in the case of the classical frequency-based method. The second part of this paper deals with a problem of estimating reliability of a series system in which the failure count data on subsystems and systems are obtained from the same time period. The multinomial distribution with the Dirichlet prior distribution is employed such that the Bayesian anomaly does not occur. A numerical example is given to compare the proposed method with the related previous methods.

2. Source of the Bayesian anomaly

In this section, we investigate the source of the Bayesian anomaly based on two examples from Bier (1994) and Philipson (1995).

2.1. Series system and exponential failure time data

Consider a series system consisting of two independent subsystems. Let X_i be the lifetime of subsystem i which follows an exponential distribution with failure rate λ_i for $i = 1, 2$. Let X be the system lifetime. Then we have $X = \min(X_1, X_2)$ and X follows an exponential distribution with failure rate $\lambda = \lambda_1 + \lambda_2$. Suppose an exponential prior distribution for λ_i, which is denoted by

$$\pi(\lambda_i) = \beta_i e^{-\beta_i\lambda_i}, \quad \lambda_i \geq 0,\ \beta_i > 0,\ i = 1, 2.$$

Suppose that the system fails at time x, and we observe that subsystem 1 fails but subsystem 2 survives. In the following, we will compare the Bayes estimate of λ obtained at the subsystem level with that obtained at the system level.

First of all, consider a problem of estimating the Bayesian system failure rate at the subsystem level. To estimate the system failure rate $\lambda = \lambda_1 + \lambda_2$, we need to estimate the subsystem failure rate λ_1 and λ_2. Given $X_1 = x$, the posterior distribution of λ_1 is obtained by

$$\pi_1(\lambda_1|X_1 = x) = \lambda_1(x + \beta_1)^2 e^{-(x+\beta_1)\lambda_1}, \quad \lambda_1 \geq 0,\ \beta_1 > 0.$$

For the squared-error loss, the Bayes estimate is the mean of the posterior distribution (Hamada et al., 2008). Thus, the Bayes estimate of λ_1 is expressed by

$$\hat{\lambda_1} = E(\lambda_1|X_1 = x) = \frac{2}{x + \beta_1}.$$

Similarly, the posterior distribution of λ_2 is given by

$$\pi_2(\lambda_2|X_2 > x) = (x + \beta_2)e^{-(x+\beta_2)\lambda_2}, \quad \lambda_2 \geq 0,\ \beta_2 > 0,$$

and the corresponding Bayes estimate of λ_2 is given by

$$\hat{\lambda_2} = E(\lambda_2|X_2 > x) = \frac{1}{x + \beta_2}.$$

Therefore, the Bayes estimate of the system failure rate obtained at the subsystem level is expressed by

$$\hat{\lambda_1} + \hat{\lambda_2} = \frac{2}{x + \beta_1} + \frac{1}{x + \beta_2}. \tag{1}$$

Second, consider a problem in which the Bayesian system failure rate is estimated at the system level. A prior distribution of λ is required for estimating the system failure rate at the system level. Philipson (1995) assumed an exponential distribution with failure rate $\beta_1 + \beta_2$ for the prior distribution of λ. In such a case, the posterior distribution of λ is a gamma distribution with parameters 2 and $\beta_1 + \beta_2 + x$ such that the Bayes estimate of the system failure rate is given by

$$E(\lambda|X = x) = \frac{2}{x + \beta_1 + \beta_2}. \tag{2}$$

In this case, however, the prior exponential distribution of λ is not consistent with that used at the subsystem level. If λ_i follows an exponential distribution with parameter β_i, then λ does not follow an exponential distribution with parameter $\beta_1 + \beta_2$. Instead, λ follows a gamma distribution with parameters 2 and β if $\beta_1 = \beta_2 = \beta$. Otherwise, if $\beta_1 \neq \beta_2$ then λ follows a mixed exponential distribution with a negative weight given by Kim (2011)

$$\pi(\lambda) = \int_0^{\lambda} \beta_1 e^{-\beta_1(\lambda-\lambda_2)}\beta_2 e^{-\beta_2\lambda_2} d\lambda_2 = \frac{\beta_1\beta_2}{\beta_2 - \beta_1}\left(e^{-\beta_1\lambda} - e^{-\beta_2\lambda}\right), \quad \lambda \geq 0,\ \beta_1, \beta_2 > 0. \tag{3}$$