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On some dependence structures for multidimensional Lévy driven moving averages

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ABSTRACT

The Lévy copula can describe the dependence structure of a multidimensional Lévy process or a multivariate infinitely divisible random variable. Suppose the Lévy copula of a multidimensional Lévy process is known. We present the Lévy copula of the Lévy measure of the moving average driven by the multidimensional Lévy process. If there exist some special dependence structures among the components of the Lévy process, we give some dependence invariance properties after the transform of the moving average.

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1. Introduction

The concept of copulas is used to describe the dependence structures of multivariate probability distributions. As an analogue to distributional copulas, the concept of Lévy copulas was introduced by Tankov (unpublished) for Lévy measures on $\mathbb{R}^d_+$ and a generalization to Lévy measures on $\mathbb{R}^d$ can be seen in Cont and Tankov (2004) and Kallsen and Tankov (2006). The Lévy copula can describe the dependence among components and does not depend on their individual laws. For a multidimensional Lévy process, the dependence of its components can be uniquely determined by its Lévy copula (see e.g. Kallsen and Tankov (2006)). Recently, Lévy driven moving averages (MAs) have drawn considerable attention in theories and applications (see e.g. Barndorff-Nielsen and Shephard (2001), Basse and Pedersen (2009) and Marquardt (2006)). If the driving process is a Lévy process, then the law of the MA is infinitely divisible (see Lemma 1). Since the dependence of components of a multivariate infinitely divisible random variable or a multidimensional Lévy process is completely characterized by the Lévy copula, this paper will focus on the Lévy copula connections between the driving Lévy process and the law of the driven MA. These connections would be useful, for example, to construct a multidimensional Lévy driven MA with a specified dependent structure. Lévy driven Ornstein–Uhlenbeck (O–U) processes are an important special case of Markov processes with jumps. The transition law and the stationary law play crucial roles in describing the properties of the O–U process, which can be treated as the law of a Lévy driven average (see Example 2).

In Section 2, we recall some related definitions and symbols of Lévy processes and Lévy copulas. In Section 3, for the MA driven by a multidimensional Lévy process, we present our general results about the Lévy copula connections between the driving Lévy process and the marginal law of the driven process. In Section 4, for some special dependence structures of components of the driving Lévy process, we provide some dependence invariance properties after the transform of the MA.

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2. Preliminaries

In this section, we shall recall a few facts on Lévy processes and Lévy measures, and some definitions related to the Lévy copula; for further details, the reader is referred to Barndorff-Nielsen and Lindner (2007), Cont and Tankov (2004), Kallsen and Tankov (2006), Sato (1999) and Tankov (unpublished).

Let $L = \{L(t), t \geq 0\}$ be a Lévy process with values in $\mathbb{R}^d$ defined on a filtered probability space $(\Omega, (\mathcal{F}_t)_{t\geq 0}, \mathcal{F}, P)$, where we assume that it has cádlág sample paths. For each $t > 0$, the random variable $L(t)$ has an infinitely divisible distribution, whose characteristic function has a Lévy–Khintchine representation

$$\mathbf{E}[e^{i\langle z, L(t)\rangle}] = \exp\left\{t\left(i\langle\gamma, z\rangle - \frac{1}{2}z^T Az + \int_{\mathbb{R}^d}\left(e^{i\langle z,x\rangle} - 1 - i\langle z, x\rangle 1_{\{|x|\leq 1\}}\right)\rho(dx)\right)\right\}, \quad z \in \mathbb{R}^d, \tag{1}$$

where $\langle\cdot,\cdot\rangle$ denotes the inner product in $\mathbb{R}^d$, $|x| = \sqrt{\langle x, x\rangle}$ denotes the Euclidean norm of $x = (x_1, \ldots, x_d)$, $\gamma \in \mathbb{R}^d$ and A is a symmetric nonnegative definite $d \times d$ matrix. The Lévy measure ρ is a measure on $\mathbb{R}^d$ satisfying $\rho(\{\mathbf{0}\}) = 0$ and $\int_{\mathbb{R}^d\setminus\{\mathbf{0}\}} 1 \wedge |x|^2\rho(dx) < \infty$. Throughout this paper, we shall work with a two-sided Lévy process $L = \{L(t), t \in \mathbb{R}\}$ constructed by taking two independent copies $\{L_1(t), t \geq 0\}$, $\{L_2(t), t \geq 0\}$ of a one-sided Lévy process and setting $L(t) = L_1(t)1_{\{t\geq 0\}} + L_2(-t-)1_{\{t<0\}}$.

In the sequel, we shall need a special interval associated with any $x \in \mathbb{R}$

$$\mathcal{I}(x) := \begin{cases}(x, \infty), & x \geq 0,\\ (-\infty, x), & x < 0.\end{cases}$$

For every Lévy measure ρ on $\mathbb{R}^d$, the *tail integral* $W = W_\rho$ can be defined as the function $W : \mathbb{R}^d \to \overline{\mathbb{R}}$ given by

$$W(x_1, \ldots, x_d) := \prod_{i=1}^{d} \operatorname{sgn}(x_i)\rho\left(\mathcal{I}(x_1) \times \cdots \times \mathcal{I}(x_d)\right) \tag{2}$$

for any $(x_1, \ldots, x_d) \in \mathbb{R}^d$. Here and hereafter, $\overline{\mathbb{R}} = (-\infty, \infty]$ and $\operatorname{sgn}(x) = 1_{\{x\geq 0\}} - 1_{\{x<0\}}$.

Let $I \subset \{1, \ldots, d\}$ be non-empty. The *I-marginal tail integral* W^I of ρ is given by

$$W^I\left((x_i)_{i\in I}\right) := \lim_{a\to 0-} \sum_{(x_i)_{i\in I^c}\in\{a,0\}^{|I^c|}} W\left(x_1, \ldots, x_d\right)\prod_{i\in I^c}\operatorname{sgn}(x_i)$$

for any $(x_i)_{i\in I} \in \mathbb{R}^{|I|}$, where $|I|$ denotes the cardinality of the set I and $I^c := \{1, \ldots, d\} \setminus I$. In fact, W^I is the tail integral of the *I-marginal Lévy measure* of ρ, which is defined by

$$\rho^I(A) := \rho\left(\{x \in \mathbb{R}^d : (x_i)_{i\in I} \in A\}\right), \quad A \subset \mathbb{R}^{|I|} \setminus \{\mathbf{0}\}. \tag{3}$$

Correspondingly, W^I is related to ρ^I by $W^I\left((x_i)_{i\in I}\right) = \prod_{i=1}^{|I|}\operatorname{sgn}(x_i)\int_{\times_{i\in I}\mathcal{I}(x_i)}\rho^I(du^I)$ for $(x_i)_{i\in I} \in \mathbb{R}^{|I|} \setminus \{\mathbf{0}\}$. To simplify notation, we denote one-dimensional margins by $W_i := W^{\{i\}}$, i.e., $W_i(x_i) = W(0, \ldots, 0, x_i, 0, \ldots, 0)$ for any $x_i \in \mathbb{R}$, $i \in \{1, \ldots, d\}$.

For $a, b \in \overline{\mathbb{R}}^d$ we write $a \leq b$ if $a_k \leq b_k$, $k = 1, \ldots, d$. In this case, let $(a, b]$ denote a right-closed left-open interval of $\overline{\mathbb{R}}^d$, i.e., $(a, b] = (a_1, b_1] \times \cdots \times (a_d, b_d]$. A function $\mathcal{C} : \overline{\mathbb{R}}^d \to \overline{\mathbb{R}}$ is called *d-increasing* if $\sum_{c\in\{a_1,b_1\}\times\cdots\times\{a_d,b_d\}}(-1)^{N(c)}\mathcal{C}(c) \geq 0$ for all $a, b \in \overline{\mathbb{R}}^d$ with $a \leq b$, where $N(c) := \#\{k : c_k = a_k, k = 1, \ldots, d\}$ with $c = (c_1, \ldots, c_d)$.

Let $\mathcal{C} : \overline{\mathbb{R}}^d \to \overline{\mathbb{R}}$ be a d-increasing function such that $\mathcal{C}(u_1, \ldots, u_d) = 0$ if $u_i = 0$ for at least one $i \in \{1, \ldots, d\}$. For any non-empty index set $I \subset \{1, \ldots, d\}$, the *I-margin of* $\mathcal{C}$ is the function $\mathcal{C}^I : \overline{\mathbb{R}}^{|I|} \to \overline{\mathbb{R}}$, defined by

$$\mathcal{C}^I((u_i)_{i\in I}) := \lim_{a\to\infty} \sum_{(u_i)_{i\in I^c}\in\{-a,\infty\}^{|I^c|}} \mathcal{C}\left(u_1, \ldots, c_d\right)\prod_{i\in I^c}\operatorname{sgn}(u_i)$$

for any $(u_i)_{i\in I} \in \overline{\mathbb{R}}^{|I|}$.

Now, we restate Definition 3.1 of Kallsen and Tankov (2006) as follows.

Definition 1. A function $\mathcal{C} : \overline{\mathbb{R}}^d \to \overline{\mathbb{R}}$ is called a *Lévy copula* if

(i) $\mathcal{C}(u_1, \ldots, u_d) \neq \infty$ for $(u_1, \ldots, u_d) \neq (\infty, \ldots, \infty)$,
(ii) $\mathcal{C}(u_1, \ldots, u_d) = 0$ if $u_i = 0$ for at least one $i \in \{1, \ldots, d\}$,
(iii) $\mathcal{C}$ is d-increasing, and
(iv) $\mathcal{C}^{\{i\}}(u) = u$ for any $i \in \{1, \ldots, d\}$, $u \in \overline{\mathbb{R}}$.

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