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Monitoring persistent change in a heavy-tailed sequence with polynomial trends

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ABSTRACT

This paper considers, for the first time, sequential monitoring against a change from $I(1)$ to $I(0)$ in a heavy-tailed sequence with polynomial trends. To detect the persistent change quickly and powerfully, a moving kernel-weighted variance ratio statistic is proposed, which is based on the sequentially updated residual process. The null distribution of the monitoring statistic and its consistency under the alternative hypothesis are proved. Simulations indicate that our procedure can achieve a good performance on a finite sample for both early change and late change. The effectiveness of the proposed procedures is well demonstrated by two sets of financial series.

1. Introduction

The presence of change-points in key macroeconomics and finance in developed economies appears to be relatively common because a myriad of political and economic factors can cause the relationships among economic variables to change over time. A typical approach in change point analysis is to execute the statistical investigation based on a fixed sample. Many papers considered fixed-sample tests for the change point, such as Csörgő and Horváh (1997), Gombay (2008), Page (1955), etc. Sequential procedures seem more useful when a decision has to be made online as new data become available. Chu, Stinchcombe, and White (1996) developed a sequential test procedure for the linear regression model. Horváth, Huskova, Kokoszka, and Steinebach (2004) detected the parameter change in the linear model sequentially using a CUSUM type test statistic. Gombay and Serban (2009) proposed a sequential test to monitor a parameter change in $AR(p)$ time series models.

In particular, there are growing evidences to indicate that economic and financial time series display changes in persistence, such as inflation rate series, short-term interest rates, government budget deficits, real output series, etc. Thus a number of test procedures have been developed to identify the changes in persistence. For a fixed sample, some well known approaches are residual based ratio tests (e.g. Carvaliere & Taylor, 2008, Kim, 2000 and Leybourne & Kim, 2003), LBI tests (Busetti & Taylor, 2004; Leybourne & Taylor, 2006) and a CUSUM of the squares-based test (Leybourne, Taylor, & Kim, 2007; Sibbertsen & Kruse, 2009). In sequential change detection, Steland (2007, 2008) studied the change from $I(1)$ to $I(0)$ in $AR(1)$ and polynomial regression models, respectively, using the kernel weighted variance ratio statistic (KWVR). Very recently,

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Chen, Tian, and Wei (2010) proposed a moving ratio monitoring scheme to detect changes in linear time series between trend stationary ($I(0)$) and difference stationary ($I(1)$) regimes.

All the aforementioned works only considered the case where variance of the sequences are finite. While an overwhelming majority of data from economics and finance, just as Guillaume (1997), Rechev and Mittnik (2000) argued, have a heavier tail than the normal variables which is preferably modeled by some κ-stable processes ($\kappa < 2$), where the tail index κ can reflect the heaviness of the data. The great advantage of these κ-stable processes is that the variance is infinite, thus, testing for changes within this sequence raises a great deal of interest in the literature. For a fixed sample with such an innovation case, Horváth and Kokoszka (2003) tested unit root using a bootstrap approximation, Han and Tian (2007a,b) investigated persistent change based on a ratio test. Chen, Tian, and Zhao (2012) considered the monitoring of changes from $I(0)$ to $I(1)$ in the infinite variance heavy-tailed observations with linear trends. But the economic time series usually show a nonlinear trends, for example, some stock price, spending index and commodity retail sales have the polynomial trends.

In this paper, we respect the fact mentioned above and, for the first time, put forward a sequential monitoring procedure designed to detect the persistence change in a heavy-tailed sequence with polynomial trends. We proposed a moving kernel-weighted variance ratio (MKWVR) monitoring procedure based on KWVR procedure and detect the change from $I(1)$ to $I(0)$. Simulation and empirical application confirm our procedure is more powerful and quickly.

The rest of this paper is organized as follows. In Section 2, we introduce the monitoring problem and show all necessary assumption. Section 3 contains the definition of sequentially update residuals, the monitoring procedure and the asymptotic results. In Section 4, we investigate the finite sample performance of our monitoring procedure and illustrate it by analyzing the changes in US inflation rate data and USD/CNY exchange rate data. Conclusion is drawn in Section 5. All the proofs of the theorems are given in Appendix.

2. Testing problem and assumptions

Assume we observe sequentially a time series $\{Y_t, t \in \mathbb{N}\}$ of real-valued observations satisfying

$$Y_t = x_t'\beta + \varepsilon_t, \quad t \in \mathbb{N}, \tag{1}$$

where $x_t = (1, t, \ldots, t^p)'$, $\beta = (\beta_0, \ldots, \beta_p)' \in \mathbb{R}$ are unknown regression coefficients and $p \in \mathbb{Z}^+$, $\varepsilon_t = \rho_t \varepsilon_{t-1} + u_t,\ t \in N$, $\rho_t \in (-1, 1]$ are unknown parameters. u_t is the stochastic part of the process which with further discussion satisfies the following assumption.

Assumption 2.1. The strictly stationary symmetrical sequence $\{u_t, t \in \mathbb{N}\}$ is in the domain of attraction of a stable law with tail index $\kappa \in (1, 2)$ and $Eu_t = 0$.

Lemma 2.1. *If Assumption 2.1 holds, then*

$$a_T{}^{-1} \sum_{i \le [Ts]} u_i \xrightarrow{d} U(s), \quad T \to \infty,$$

where $a_T = \inf\{x : P(|u_t| > x) \le T^{-1}\}$ and the random variable $U(s)$ is κ-stable Lévy process in $[0, 1]$. The notation $\xrightarrow{d}$ stands for convergence in distribution.

Remark 2.1. This result can be found in Kokoszka and Wolf (2004) and Resnick (1986). The exact definition of the Lévy process appearing in Lemma 2.1 is not needed in the following, but we recall that the quantities a_T can be represented as $a_T = T^{1/\kappa} L(T)$ for some slowly varying function $L(\cdot)$.

We consider the following change-point testing problem. The null hypothesis,

$$H_0 : \rho_t = 1 \quad \text{for all } t,$$

states the error terms of the regression model form a random walk; i.e. are integrated of order 1. The alternative hypothesis

$$H_1 : \rho_t = 1, \quad t < q,\ \rho_t = \rho,\ t \ge q,\ |\rho| < 1$$

specifies that there exists a change-point such that the subseries $\varepsilon_t : t \ge q$ satisfies stationary equations. It is important to note that the method proposed in this article does not require any specification of an alternative.

Our monitoring procedure is based on the following assumption,

Assumption 2.2. Assume $\rho_t = 1$ for $t = 1, \ldots, [T\tau]$, $\tau \in (0, 1)$.

This assumption is similar to the non-contamination assumption in Horváth et al. (2004) which monitor structure change in linear regression models. $[T\tau]$ also be called historical sample size. We start from the $[T\tau] + 1$ sample and sequentially detect change until the time where the procedure gives a signal or the time horizon T.