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# Discrete-time $GI^X/Geo/1/N$ queue with negative customers and multiple working vacations

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ABSTRACT

Using the supplementary variable and the embedded Markov chain method, we consider a discrete-time batch arrival finite capacity queue with negative customers and working vacations, where the RCH killing policy and partial batch rejection policy are adopted. We obtain steady-state system length distributions at pre-arrival, arbitrary and outside observer's observation epochs. Furthermore, we consider the influence of system parameters on several performance measures to demonstrate the correctness of the theoretical analysis.



## 1. Introduction

Recently, queueing systems with negative customers have been studied extensively, as they have been greatly motivated by some practical applications such as computer, neural networks, manufacturing systems and communication networks (see Artalejo, 2000). Negative customers can be thought of as viruses or commands to delete some transactions in a computer network or a database, inhibitor and synchronization signals in neural and high speed communication network; see Chao, Miyazawa, and Pinedo (1999).

Queues with negative arrivals, called $G$-queues, were first introduced by Gelenbe (1989). When a negative customer arrives at the queue, it immediately removes one or more positive customers if present. Negative arrivals have no effect if the system is empty. A negative customer can remove positive customers in the queue (or being served) according to different specified killing disciplines: (i) RCH (removal of the customer from the head of the queue); (ii) RCE (removal of the customer from the end of the queue). Since 1989, there have been many studies on queues with negative customers; readers may refer to Boucherie and Boxma (1996), Chakka and Harrison (2001), Harrison, Patel, and Pitel (2000), Harrison and Pitel (1993, 1995, 1996), Li and Zhao (2004) and Wu, Liu, and Peng (2009).

Although many continuous-time queueing models with negative arrivals have been studied extensively in the past years, their discrete-time counterparts received very little attention in the literature. It is well known that discrete-time queueing models are more suitable for application to digital communication systems, including mobile and broad integrated services digital networks (B-ISDN) based on asynchronous transfer mode (ATM) technology, because these protocols are operated

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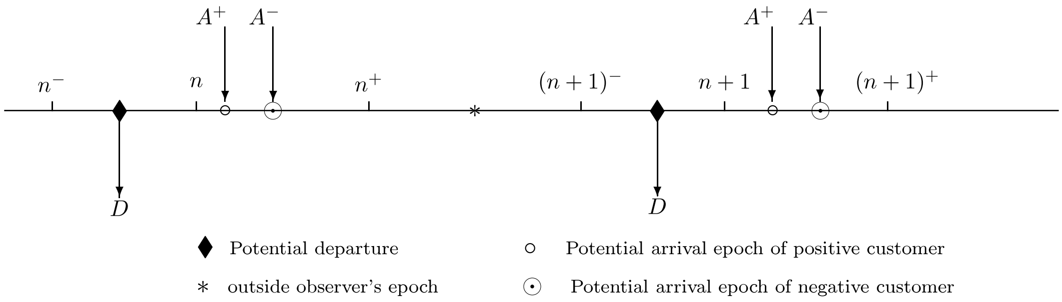


**Fig. 1.** Various time epochs in an early arrival system.

by a unit of slot which is an equal time interval (see Bruneel & Kim, 1993). The work about negative customers in discrete-time can be found in Atencia and Moreno (2004a,b, 2005), where the authors considered the single-server discrete-time queue with negative arrivals and various killing disciplines caused by the negative customers, Wang and Zhang (2009), where they presented a discrete-time single-server retrial queue with geometrical arrivals of both positive and negative customers. Other more work on negative customers can be found in Do (2011), where Do (2011) presented a bibliography on G-networks from 1989 to 2010.

Recently, queueing systems with working vacations have been widely used in the performance analysis of communication systems, in which the server renders service to the customers with a lower service rate during vacation period. Working vacation policy has practical application background in optimal design of the system. When the number of customers in the system is relatively few, we set a lower speed operating period in order to economize operating cost and energy consumption. The researches about working vacation queueing models with infinite buffer size can be found in Baba (2005), Gao and Liu (2013), Kim, Choi, and Chae (2003), Li, Liu, and Tian (2010), Li and Tian (2007), Li, Tian, and Liu (2007), Li, Tian, and Ma (2008), Li, Tian, Zhang, and Luh (2009), Liu, Xu, and Tian (2007), Servi and Finn (2002), Wu and Takagi (2006), Yi, Kim, Choi, and Chae (2007) and the references therein.

Though the working vacation queueing models with infinite buffer size have been studied extensively in the past years, many a time there is also need for finite buffer size. Queues with finite buffer space are more realistic in real life situations than queues with infinite buffer space as it is used to store arrived customers if server is busy. To the best of our knowledge, the works about general input working vacation queueing model with finite buffer size can be found in Banik, Gupta, and Pathak (2007), Goswami and Mund (2010), Goswami and Vijaya Laxmi (2010), Yu, Tang, and Fu (2009) and Yu, Tang, Fu, and Pan (2011), where Banik et al. (2007) discussed the $GI/M/1/N$ queue with multiple working vacations, Yu et al. (2009) presented the $GI^{[x]}/M^b/1/L$ queue with multiple working vacations and partial batch rejection, Goswami and Vijaya Laxmi (2010) analyzed the $GI^{[x]}/M/1/N$ queue with single working vacations and partial batch rejection, a finite buffer size discrete-time multiple working vacation queue was considered by Goswami and Mund (2010) and Yu et al. (2011) have introduced changeover time into the working vacation. Nevertheless, discrete-time finite buffer $G$-queues taking the working vacation policy into account have not been studied up to now. Thus, the contribution of this work is to analyze a discrete-time finite buffer queue with multiple working vacations and two types of customers, positive and negative, in accordance with the RCH.

The rest of this paper is organized as follows. In Section 2, we give the discrete time queueing model. In Section 3, we analyze the model and obtain steady state distributions at arbitrary, pre-arrival and outside observer's observation epochs. Various performance measures and numerical examples are presented in Section 4.

## 2. Model formulations

Thereinafter, we denote $\bar{x} = 1 - x$ for any real number $x \in (0, 1)$. The $GI^X/Geo/1/N$ queue with negative customers and multiple working vacations we considered here is an early arrival system that is, a potential arrival can only take place in $(n, n^+)$ and a potential departure can only take place in $(n^-, n)$. We assume that the beginning and ending of vacations occurs at the instant $n$. Arriving customers are queued according to the first-come, first-served (FCFS) discipline. The server can serve only one customer at a time. Various stochastic processes involved in the system are independent of each other. The various time epochs at which events occur are depicted in Fig. 1.

The detailed description of the model is given as follows:

(1) Positive customers arrive in batches of random size $X$ with probability mass function (p.m.f.) $P(X = j) = \chi_j, j = 1, 2, \ldots,$ and mean $E[X] = \mu_X$. The inter-arrival times $A$ of two successive batches of positive customers are independently identically distributed (i.i.d.) random variables with common p.m.f. $P(A = i) = a_i,\ i \geq 1$, corresponding p.g.f. $\widetilde{A}(z) = \sum_{i=1}^{\infty} a_i z^i$ and mean inter-arrival time $\lambda^{-1} = \widetilde{A}'(1)$, where $\widetilde{A}'(1)$ is the first derivative of $\widetilde{A}(z)$ with respect to $z$ at $z = 1$.

(2) Inter-arrival times $B$ of negative customers are independent and geometrically distributed random variable with the following geometric distribution:

$$P(B = j) = \eta\bar{\eta}^{j-1}, \quad j \geq 1,\ 0 \leq \eta < 1.$$