Journal of the Korean Statistical Society 42 (2013) 543–564

Contents lists available at ScienceDirect

Journal of the Korean Statistical Society

journal homepage: www.elsevier.com/locate/jkss

On a perturbed MAP risk model under a threshold dividend strategy

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ARTICLE INFO

Article history:
Received 6 November 2012
Accepted 14 March 2013
Available online 10 April 2013

AMS 2000 subject classifications:
60J65
62P05

Keywords:
Markovian arrival process
Gerber–Shiu function
Threshold dividend strategy
Discounted dividend payments
Matrix renewal equation

ABSTRACT

In this paper, we consider a perturbed risk model where the claims arrive according to a Markovian arrival process (MAP) under a threshold dividend strategy. We derive the integro-differential equations for the Gerber–Shiu expected discounted penalty function and the moments of total dividend payments until ruin, obtain the analytical solutions to these equations, and give numerical examples to illustrate our main results. We also get a matrix renewal equation for the Gerber–Shiu function, and present some asymptotic formulas for the Gerber–Shiu function when the claim size distributions are heavy-tailed.

1. Introduction

In this paper, we consider a continuous-time surplus process perturbed by a diffusion where the claims arrive according to a Markovian arrival process (MAP).

Let $\{N(t),\ t \geq 0\}$ be the claim number process and $\{J(t),\ t \geq 0\}$ be an underlying homogeneous continuous time Markov chain (CTMC) with finite state space $E = \{1, 2, \ldots, n\}$ affecting the claim arrivals. The evolution of the bivariate Markov process $\{N(t), J(t),\ t \geq 0\}$ on the space $\mathbb{N} \times \{1, 2, \ldots, n\}$ is governed by the matrices $\mathbf{D_0}$ and $\mathbf{D_1}$, where $\mathbf{D_0} = [D_{0,ij}]_{i,j=1}^n$ and $\mathbf{D_1} = [D_{1,ij}]_{i,j=1}^n$ are such that

$$0 \leq D_{0,ij} < \infty,\ i \neq j,\ D_{0,ii} < 0,\ 0 \leq D_{1,ij} < \infty, \quad \sum_{j=1}^{n} (D_{0,ij} + D_{1,ij}) = 0.$$

$D_{0,ij}$ corresponds to the instantaneous rate of transition from state i to state j ($j \neq i$) in E without an accompanying claim, and $D_{1,ij}$ corresponds to the instantaneous rate of transition from state i to state j (possibly $j = i$) in E with an accompanying claim. The CTMC $\{J(t), t \geq 0\}$ is assumed to be irreducible with initial distribution $\vec{\alpha} = (\alpha_1, \alpha_2 \ldots, \alpha_n)^\top$ and intensity matrix $\mathbf{D_0} + \mathbf{D_1}$. Under these assumptions, the bivariate Markov process $\{N(t), J(t), t \geq 0\}$ is called a MAP, with representation MAP$(\vec{\alpha}, \mathbf{D_0}, \mathbf{D_1})$, we also say that the claim arrival process is a MAP. We remark that the MAP risk model contains the classical compound Poisson risk model, Markov-modulated risk model and Sparre Andersen risk model with phase-type inter-claims as special cases, see Ren and Li (2009).

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http://dx.doi.org/10.1016/j.jkss.2013.03.004

To account for small fluctuation in the surplus level, as that in Dufresne and Gerber (1991), we assume that the surplus process is perturbed by a Brownian motion with drift 0 and infinitesimal variance σ_i^2 whenever the CTMC $\{J(t),\ t \geq 0\}$ is in state i. Then the perturbed MAP risk model is defined as

$$U(t) = u + c_1 t - \sum_{i=1}^{N(t)} X_i + \int_0^t \sigma_{J(s)} \mathrm{d}B(s), \quad t \geq 0, \tag{1.1}$$

where $c_1 > 0$ is the constant premium rate. X_i represents the size of the ith claim and $\{B(t),\ t \geq 0\}$ is an independent standard Brownian motion. For a transition of $\{J(t),\ t \geq 0\}$ from state i to state j at the time of a claim, we assume that the accompanying claim size has density function $f_{ij}(x)$, distribution function $F_{ij}(x)$, Laplace transform $\hat{f}_{ij}(s) = \int_0^\infty f_{ij}(x) e^{-sx} \mathrm{d}x$ and finite mean μ_{ij}. It is clear that the random variables $\{X_i,\ i \geq 1\}$ are not independent in general, but conditional on $\{J(t),\ t \geq 0\}$, they are independent.

The MAP has received much attention in the queuing literature, see Section VII in Asmussen and Albrecher (2010), or Asmussen (2003) for details. In the last few years, the MAP risk model has received a lot of attention. Badescu et al. (2005); Badescu, Breuer, Drekic, Latouche, and Stanford (2005) studied the ruin probability and the joint distribution of the surplus before and after ruin. Ahn and Badescu (2007) studied the discounted penalty function. Due to the practical importance, dividend strategies have been one of the main research problems in ruin theory today, Avanzi (2009) presented excellent reviews about all kinds of risk models under dividend strategies. For the MAP risk model, Ahn, Badescu, and Ramaswami (2007) studied the barrier dividend, and Badescu, Drekic, and Landriault (2007a,b) studied the threshold dividend and multi-threshold dividend strategies, respectively. In these papers, the assumption on the phase-type claim size distribution is important, so that the risk model can be connected to the fluid flow model.

Compared to those results, relying on a purely analytic approach, a variety of theoretical results for the general claim size distribution are obtained, for example, Badescu (2008) derived the integro-differential equation for the discounted penalty function and solved at $u = 0$ for the MAP risk model with no dividend involved, Ren (2009) further derived a matrix expression for the Laplace transform of the first time that the surplus process reaches a given target from u for the perturbed MAP risk model, Cheung and Landriault (2009) considered the perturbed MAP risk model under barrier dividend, Chen (2009) considered the MAP risk model under multi-threshold dividend, and recently, Cheung and Landriault (2010) studied a generalized penalty function for the MAP risk model, Zhang, Yang, and Yang (2011) considered the absolute ruin in a MAP risk model with debit interest.

In this paper, we consider the perturbed MAP risk model under the threshold dividend strategy. With a dividend barrier at level $b > 0$, the surplus process with initial surplus $U_b(0) = u$ follows the dynamics

$$\mathrm{d}U_b(t) = \begin{cases} c_1 \mathrm{d}t - \mathrm{d}\left(\sum_{i=1}^{N(t)} X_i\right) + \sigma_{J(t)} \mathrm{d}B(t) & 0 \leq U_b(t) < b \\ c_2 \mathrm{d}t - \mathrm{d}\left(\sum_{i=1}^{N(t)} X_i\right) + \sigma_{J(t)} \mathrm{d}B(t) & U_b(t) \geq b, \end{cases} \tag{1.2}$$

in which $c_2 = c_1 - d$, that is to say, dividends are paid continuously at a constant rate d $(0 < d \leq c_1)$ whenever the surplus is above $b > 0$. As that in Cheung and Landriault (2009), we assume that the net profit condition is fulfilled, i.e.,

$$\sum_{i=1}^n \pi_i \sum_{j=1}^n D_{1,ij} \mu_{ij} < c_2,$$

where $\vec{\pi} = (\pi_1, \ldots, \pi_n)$ are the stationary probability of the CTMC $\{J(t),\ t \geq 0\}$, which are the solutions of the following system of linear equations:

$$\sum_{j=1, j\neq i}^n \pi_j (D_{0,ji} + D_{1,ji}) = -\pi_i (D_{0,ij} + D_{1,ij}), \quad i \in E, \qquad \sum_{j=1}^n \pi_j = 1.$$

For risk model (1.2), let $T_b = \inf\{t \geq 0, U_b(t) \leq 0\}$ ($T_b = \infty$ if the set is empty) be the ruin time, $\psi(u, b) = P(T_b < \infty | U_b(0) = u)$ be the ruin probability, $U_b(T_b-)$ be the surplus immediately before ruin, and $U_b(T_b)$ be the deficit at ruin. To describe these and many other risk quantities, we introduce the Gerber–Shiu expected discounted penalty function (see Gerber & Shiu, 1998), defined as

$$\Phi(u, b) = E[e^{-\delta T_b} \omega(U_b(T_b-), |U_b(T_b)|) I(T_b < \infty) | U_b(0) = u],$$

where $\delta > 0$, $\omega(x, y),\ x \geq 0,\ y \geq 0$, is a non-negative real function such that the expectation exists with $\omega(0, 0) = 1$, and $I(C)$ is the indicator function of a set C.

Given the initial surplus is u and the initial MAP state is $i \in E$, define

$$\Phi_{ij}(u, b) = E[e^{-\delta T_b} \omega(U_b(T_b-), |U_b(T_b)|) I(T_b < \infty, J(T_b) = j) | U_b(0) = u, J(0) = i],$$