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Random central limit theorems for linear processes with weakly dependent innovations

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ABSTRACT

Random central limit theorems (CLTs) are established for a linear process driven by a strictly stationary ψ-weakly dependent process as well as for the ψ-weakly dependent process itself, whose dependence structure was introduced by Doukhan and Louhichi [Doukhan, P., & Louhichi, S. (1999). A new weak dependence condition and applications to moment inequalities. Stochastic Processes and their Applications, 30 84, 313–342] to generalize mixings and other dependence. Random CLTs are established for partial sums and sample autocovariances of the ψ-weakly dependent process and the linear process under absolute summability.

1. Introduction

The ψ-weak dependence proposed by Doukhan and Louhichi (1999) and simultaneously by Bickel and Bühlmann (1999) is a natural generalization of the widely accepted mixing dependence. A main advantage is that the class of ψ-weakly dependent processes is much wider than that of mixing processes containing lots of pertinent examples. The ψ-weak dependence includes essentially all classes of weakly dependent stationary processes such as mixing, association, Gaussian sequences and Bernoulli shifts of interest in statistics under natural conditions on the process parameters. For example, Ango Nze, Bühlmann, and Doukhan (2002) and Dedecker et al. (2007) discussed many nonlinear time series processes such as GARCH, bilinear, threshold AR processes in terms of ψ-weak dependence.

We study asymptotics for a stationary ψ-weakly dependent process $\{\varepsilon_t\}$ and for a linear process $\{X_t : t \in \mathbb{Z}\}$ given by

$$X_t = \sum_{j=0}^{\infty} a_j \varepsilon_{t-j} \tag{1}$$

where $\{a_j\}$ is a sequence of real numbers with $\sum_{j=0}^{\infty} |a_j| < \infty$. We establish random CLTs for the basic statistics of partial sums and sample autocovariances: $\{\sum_{i=1}^{n} \varepsilon_i, \sum_{i=1}^{n} X_i\}$ and $\{\sum_{i=1}^{n} \varepsilon_i \varepsilon_{i+h}/n, \sum_{i=1}^{n} X_i X_{i+h}/n,\ h = 0, 1, 2, \ldots\}$.

Many authors have investigated CLTs for partial sums under diverse situations. For the case of i.i.d. innovations $\{\varepsilon_t\}$, Hosking (1996), Phillips and Solo (1992) and Yokoyama (1995) among others studied CLTs for the partial sums $\sum_{i=1}^{n} X_i$.

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Juodis and Rackaukas (2007) established a CLT for self-normalized sums of a linear process, while Mynbaev (2009) dealt with a CLT for weighted sums of a linear process. For dependent innovations, Peligrad and Utev (1997) studied CLTs of linear processes under several well-known mixings and associated sequences, and their results were extended to ψ-weakly dependent processes by Coulon-Prieur and Doukhan (2000). Wu and Min (2005) and Wu and Woodroofe (2004) considered CLTs of linear processes under a wide class of dependent innovations whose dependence structure involves conditional moments. Peligrad and Utev (2006) established a CLT for linear processes with dependent innovations including martingales and mixingale type of assumptions.

On the other hand, as for the CLTs of sample autocovariances/autocorrelations, Romano and Thombs (1996) investigated large-sample behavior of sample autocorrelations of a weakly dependent stationary process including linear processes along with resampling methods, and Chung (2002) and He (1996) considered linear processes with martingale difference innovations. Also, Wu and Min (2005) studied CLTs for the sample autocovariances under the same dependence condition adopted for the CLT study of partial sums. Horvath and Kokoszka (2008), Hosking (1996) and Kokoszka and Taqqu (1996) and obtained asymptotic distributions of the sample autocovariances of long-range dependent linear processes.

Concerning random CLTs of the partial sum $\sum_{i=1}^{n} \varepsilon_i$, we refer to Blum, Hanson, and Rosenblatt (1963) and Reyni (1960) for i.i.d. assumption on $\{\varepsilon_i\}$, and Prakasa Rao (1969) for martingale difference ϕ-mixing. For the linear process as in (1), Fakhre-Zakeri and Farshidi (1993) and Fakhre-Zakeri and Lee (1992) developed a random CLT of $\sum_{i=1}^{n} X_i$ with i.i.d. innovations $\{\varepsilon_i\}$. Lee (1997) established random CLTs for $\sum_{i=1}^{n} \varepsilon_i$ and $\sum_{i=1}^{n} X_i$ under strong mixing $\{\varepsilon_i\}$.

We first extend the results of the random CLTs for the partial sums of ε_i and X_i, established by Lee (1997) for strong mixing errors, to those for more general ψ-weakly dependent errors. Additional generality is achieved for a more general condition on the random indices. Secondly we give the large-sample behaviors for the sample autocovariances of ε_i and X_i. Both ordinary CLTs and random CLTs are investigated for the sample autocovariances. The asymptotic distributions of the partial sums and the sample autocovariances of the linear process X_i are discussed under absolute summability.

This paper is organized as follows. Section 2 describes the ψ-weak dependence condition of stationary processes. In Section 3 we give assumptions and state our main results of the random CLTs for the partial sums and the sample autocovariances. Technical proofs are given in Section 4.

2. ψ-weak dependence

To define the notion of the ψ-weak dependence, we introduce some classes of functions. Let $\mathbb{L}^\infty = \bigcup_{n=1}^{\infty} \mathbb{L}^\infty(\mathbb{R}^n)$, the set of real-valued and bounded functions on $\mathbb{R}^n$ for $n = 1, 2, \dots$. Consider a function $g : \mathbb{R}^n \to \mathbb{R}$ where $\mathbb{R}^n$ is equipped with its l_1-norm (i.e. $\|(x_1, \dots, x_n)\|_1 = |x_1| + \cdots + |x_n|$) and define the Lipschitz modulus of g as follows:

$$\mathrm{Lip}(g) = \sup_{x \neq y} \frac{|g(x) - g(y)|}{\|x - y\|_1}.$$

Let

$$\mathcal{L} = \bigcup_{n=1}^{\infty} \mathcal{L}_n \quad \text{where } \mathcal{L}_n = \{g \in \mathbb{L}^\infty(\mathbb{R}^n);\ \mathrm{Lip}(g) < \infty,\ \|g\|_\infty \le 1\}.$$

The class $\mathcal{L}$ is sometimes used together with the following functions $\psi = \psi_0, \psi_1, \psi_2, \eta, \kappa$ and λ, which yield notions of weak dependence appropriate to describe various examples of models: $\psi_0(g, h, n, m) = 4\|g\|_\infty \|h\|_\infty$, $\psi_1(g, h, n, m) = \min(n, m)\mathrm{Lip}(g)\mathrm{Lip}(h)$, $\psi_2(g, h, n, m) = 4(n+m) \min\{\mathrm{Lip}(g), \mathrm{Lip}(h)\}$, $\eta(g, h, n, m) = n\mathrm{Lip}(g)+m\mathrm{Lip}(h)$, $\kappa(g, h, n, m) = nm\mathrm{Lip}(g)\mathrm{Lip}(h)$, $\lambda(g, h, n, m) = n\mathrm{Lip}(g) + m\mathrm{Lip}(h) + nm\mathrm{Lip}(g)\mathrm{Lip}(h)$, for functions g and h defined on $\mathbb{R}^n$ and $\mathbb{R}^m$ respectively. See Dedecker et al. (2007) and Doukhan and Neumann (2007).

Definition 2.1 (*Doukhan & Louhichi, 1999*)**.** The sequence $\{\varepsilon_t\}_{t\in\mathbb{Z}}$ is called $(\theta, \mathcal{L}, \psi)$-*weakly dependent* (simply, ψ-*weakly dependent*), if there exists a sequence $\theta = (\theta_r)_{r\in\mathbb{Z}}$ decreasing to zero at infinity and a function ψ with arguments $(g, h, n, m) \in \mathcal{L}_n \times \mathcal{L}_m \times \mathbb{N}^2$ such that for n-tuple $(i_1, \dots, i_n)$ and m-tuple $(j_1, \dots, j_m)$ with $i_1 \le \cdots \le i_n < i_n + r \le j_1 \le \cdots \le j_m$, one has

$$|\mathrm{Cov}(g(\varepsilon_{i_1}, \dots, \varepsilon_{i_n}), h(\varepsilon_{j_1}, \dots, \varepsilon_{j_m}))| \le \psi(g, h, n, m)\theta_r.$$

According to Doukhan and Louhichi (1999), strong mixing is ψ_0-weakly dependent, associated sequences are ψ_1-weakly dependent, and Bernoulli shifts and Markov processes are ψ_2-weakly dependent.

There have been recently many studies on ψ-weakly dependent processes. Doukhan and Neumann (2008) provided an overview of classes of processes possessing the properties of ψ-weak dependence and described important probabilistic results. Doukhan and Louhichi (1999) established the ordinary CLTs of the partial sums of the weakly dependent sequences and moment inequalities such as Marcinkiewicz–Zygmund, Rosenthal and exponential inequalities. Doukhan and Louhichi (2001) analyzed the behavior of a standard kernel density estimator in view of the asymptotics of high order losses, asymptotic distributions and uniform almost sure behaviors. See Dedecker and Prieur (2004), Doukhan and Neumann (2007) and Kallabis and Neumann (2006) for several useful inequalities under the ψ-weak dependence.