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Monitoring persistence change in infinite variance observations

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ABSTRACT

In this paper, we adopt a kernel-weighted variance ratio statistic to monitor persistence change in infinite variance observations. We focus on a $I(0)$ to $I(1)$ regime switch for sequences in the domain of attraction of a stable law and local-to-finite variance sequences. The null distribution of the monitoring statistic and its consistency under alternative hypothesis are proved. In particular, a bootstrap approximation is proposed to determine the critical values for the derived asymptotic distribution depends on the unknown tail index. The small sample performance of the proposed monitoring procedures are illustrated by both simulation and application to Sweden/US foreign exchange rate data.

1. Introduction

During the past two decades, there is a growing body of evidence to suggest that economic and financial time series display changes in persistence. This has been an issue of substantial empirical interest, especially concerning inflation rate series, short-term interest rates, government budget deficits and real output. Fixed-sample tests for such change point problem have been well studied in the literature. These include residual based ratio tests (e.g. Kim (2000), Leybourne and Kim (2003)), and LBI tests (e.g. Busetti and Taylor (2004), Leybourne and Taylor (2006)). As a general discussion for these methods we refer the reader to Perron (2006). More recently, Leybourne, Taylor, and Kim (2007) proposed a CUSUM of squares-based test. Carvaliere and Taylor (2008) considered a persistence change test under the non-stationary volatility innovation case; Sibbertsen and Kruse (2009) studied the long-range dependence innovation case. Sibbertsen (2004), Hassler and Scheithauer (2009) studied change in persistence within a fractionally integrated framework. While many economic data are observed sequentially and need to be analyzed on-line, Steland (2007) and Chen, Tian, and Wei (2010) studied the persistence change monitoring problem in on-line data.

All of the works above are concentrated on the case where variances of the sequences are finite. Guillaume et al. (1997), and Rechev and Mittnik (2000) have argued that many types of data from economics and finance have the same character: a heavier tail than the normal variables, and it is more suitable to model these heavy-tailed data by some κ-stable processes, where the tail index κ can reflect the heaviness of the data. The variance of κ-stable processes is infinite when $\kappa < 2$. Therefore, testing for change within this sequence raises a great deal of interest in the literature. For a fixed sample with such an innovation case, Horváth and Kokoszka (2003) tested unit root, Han and Tian (2007a,b) investigated persistence

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change based on a ratio test. Recently, Chen and Tian (2010) considered an on-line monitoring problem of stationarity in a heavy tailed sequence.

This paper focus on the $I(0)$ to $I(1)$ persistence change monitoring problem for observations in the domain of the attraction of a stable law. We also consider a local-to-finite variance case, which is helpful to highlight the behavior of the test in borderline or near borderline cases between finite and infinite variance. Our monitoring procedure is based on a kernel weighted variance ratio statistic. The kernel in our monitoring statistic is applied to ensure that past partial sums have smaller weights than more recent ones. In order to avoid estimating the tail index, we also propose a bootstrap method to determine the critical values. We show via simulation and empirical application that the proposed monitoring procedure performs well. In many cases, the change point (if it occurs) can be detected very early, and it is not necessary to wait until the time horizon.

The rest of the paper is organized as follows. Section 2 introduces the model and some necessary assumptions. In Section 3 we introduce our monitoring procedure, and derive its asymptotic properties both for κ-stable sequences and local-to-finite variance sequences. The bootstrap method also will be provide in this section. In Section 4 we use Monte Carlo methods to investigate the finite sample performance of our monitoring procedure and illustrate it by analyzing Sweden/US foreign exchange rate data. Section 5 concludes the paper. All technical proofs of theoretical results are gathered in Section 6.

2. Model and assumptions

Let $y_1, y_2, \ldots,$ be an observed time series that can be decomposed as

$$y_t = \mu_t + \varepsilon_t, \tag{1}$$

$$\varepsilon_t = \rho_t \varepsilon_{t-1} + e_t, \quad t = 1, 2, \ldots \tag{2}$$

where $\mu_t = E(y_t) = \delta^{\mathrm{T}} d_t$ is a deterministic component modeled as a linear combination of a vector of nonrandom regressors d_t. This paper concentrates on $d_t = 0$, $d_t = 1$ and $d_t = (0, 1)^{\mathrm{T}}$ three different typical components. e_t is the stochastic part of the process which with further discussion satisfies the following assumption.

Assumption 2.1. The strictly stationary symmetrical sequences e_t are in the domain of attraction of a stable law with tail index $\kappa \in (1, 2)$ and $Ee_t = 0$.

Lemma 2.1. *If Assumption 2.1 holds, then*

$$\left(a_T^{-1} \sum_{t=1}^{[T\tau]} e_t, a_T^{-2} \sum_{t=1}^{[T\tau]} e_t^2 \right) \xrightarrow{d} (U_1(\tau), U_2(\tau)), \tag{3}$$

where T denotes the sample size,

$$a_T = \inf\{x : P(|e_t| > x) \le T^{-1}\},$$

and the random variable $U_1(\tau)$ is κ-stable and $U_2(\tau)$ is a $\kappa/2$-stable Lévy process in [0,1]. The notation $\xrightarrow{d}$ stands for convergence in distribution.

Remark 2.1. This result can be found in Resnick (1986) and Kokoszka and Wolf (2004). The exact definition of the Lévy process $(U_1(\tau), U_2(\tau))$ appearing in Lemma 2.1 is not needed in the following, but we recall that the quantities a_T can be represented as $a_T = T^{1/\kappa} L(T)$ for some slowly varying function L.

Within the model (1), the process y_t is $I(0)$ if $|\rho_t| < 1$, the process y_t is $I(1)$ if $|\rho_t| = 1$. We are interested in the following change point problem: observing sequence $y_1, y_2, \ldots,$ and detecting whether the new coming observation leads the available stationary regime changes to a nonstationary one, namely, we want to test the null hypothesis

$$H_0 : y_t \sim I(0), \quad t = 1, 2, \ldots \tag{4}$$

against the alternative hypothesis

$$H_1 : y_t \sim I(0), \quad t = 1, \ldots, k^*,$$
$$y_t \sim I(1), \quad t = k^* + 1, k^* + 2, \ldots. \tag{5}$$

Since a monitoring procedure will stop sooner or later in practice, we set the largest monitoring sample size $T < \infty$ as known. In other words, we just consider the "close ended" retrospective test. Our monitoring procedure relies on the following assumption:

Assumption 2.2. Suppose that $y_t \sim I(0),\ t = 1, \ldots, N,$ for some $0 < N < T$.

Remark 2.2. This assumption is similar to the "noncontemporary assumption" of Horváth et al. (2004) who monitors coefficient changes in linear models. For our monitoring procedure is a "close ended" test, we set N is a proportion of sample size T, namely, N equals to the start time of monitoring in Steland (2007). Simulations in Section 4 indicate that $N = 0.3T$ is a reasonable choice in our monitoring procedure.