Journal of the Korean Statistical Society 40 (2011) 161–172

Contents lists available at ScienceDirect

Journal of the Korean Statistical Society

journal homepage: www.elsevier.com/locate/jkss

Constancy test for FARIMA long memory processes

Jiyeon Lee [a], Okyoung Na [b,*], Sangyeol Lee [a]

[a] Department of Statistics, Seoul National University, Seoul, 151-747, Republic of Korea

[b] Applied Information Statistics, Kyonggi University, Suwon, 443-760, Republic of Korea

ARTICLE INFO

Article history:
Received 21 April 2010
Accepted 8 September 2010
Available online 15 October 2010

AMS 2000 subject classifications:
primary 62M10
secondary 62G10
62G20

Keywords:
Long memory process
FARIMA models
Change point
Cusum test
QMLE

ABSTRACT

In this paper, we consider the test for constancy of parameters in long memory fractional autoregressive integrated moving average (FARIMA or ARFIMA) models. We construct the cusum test on the basis of the quasi-log-likelihood estimator (QMLE) and derive its limiting null distribution. Simulation results are provided for illustration.

1. Introduction

The phenomenon of long memory or long range dependence has been widely observed in many scientific areas. In order to incorporate this phenomenon, various stochastic models are developed by researchers: see, for example, Beran (1992) and Cox (1984). Among many methodologies to analyze long memory processes, the FARIMA model approach, proposed by Granger and Joyeus (1980) and Hosking (1981), has long been popular in diverse statistical applications, especially in the field of finance: see Baillie (1996), Beran (1995), Henry (2002) and Robinson (2003), and the references therein. All the aforementioned studies on the inference in FARIMA models assume that the long memory parameter of time series remains constant over time. However, the long range dependence structure changes over time in some situations. For instance, we can see this phenomenon in the Nile river data (cf. Beran and Terrin (1996)), the US inflation data (cf. Kumar and Okimoto (2007) and Sibbertsen and Kruse (2009)).

There exist many preceding studies pertaining to the test for the change of the long memory parameter. For instance, Beran and Terrin (1996) pointed out that the long range dependence structure appears to change over time in some time series data and suggested a procedure to test for the stability of the long memory parameter. Their test is based on a frequency domain approach that uses Whittle's estimates, and similar tests have been proposed by Horváth (2001) and Horváth and Shao (1999). Recently, unlike these authors, Ling (2007) used a time domain approach based on the Wald test and applied it to long memory FARIMA models. Meanwhile, Sibbertsen and Kruse (2009) studied the cusum of squares test for detecting the change of the long memory parameter.

In this study, we consider the cusum test designed by Lee, Ha, Na, and Na (2003) for the constancy of the FARIMA parameters. Their procedure is to compare the estimates of the target parameter obtained based on a growing number of

* Corresponding author.
E-mail address: okna1230@gmail.com (O. Na).

1226-3192/$ – see front matter
doi:10.1016/j.jkss.2010.09.001

observations with the estimate obtained based on the whole observations. Below, we illustrate the procedure of constructing the cusum test when the target parameter θ is 1-dimensional. Let $\hat{\theta}_k,\ k = 1, \ldots, n$, be the estimates based on the observations up to time k and define

$$U_{n,k} = \frac{k}{\sqrt{n\hat{\omega}_n^2}}(\hat{\theta}_k - \hat{\theta}_n), \quad k = 1, 2, \ldots, n, \tag{1.1}$$

where $\hat{\omega}_n^2$ is an estimate of the asymptotic variance ω^2 of $\hat{\theta}_n$: usually, under the null hypothesis of no changes, $\sqrt{n}(\hat{\theta}_n - \theta)$ has a normal distribution as its limiting law. If there are no changes in the parameter θ over the sample period, $U_{n,[ns]},\ s \in [0, 1]$, behaves asymptotically the same as a Brownian bridge. In contrast, if there are parameter changes in the series, the maximum of the absolute value of $U_{n,k}$ gets larger. For instance, if a single change occurs at k^*, the plot of $|U_{n,k}|$ against k has a peak around the change point k^*. If $\max_{1\le k\le n} |U_{n,k}|$ is greater than a preassigned threshold, the null hypothesis is rejected. In this case, $\hat{k} = \mathrm{argmax}_k |U_{n,k}|$ is taken as an estimate of the change point.

This paper is organized as follows. In Section 2, we introduce the cusum test in long memory FARIMA models. The cusum test statistic is constructed on the basis of the quasi-log-likelihood estimator. It is shown that under regularity conditions, the limiting null distribution of the test is the sup of the squares of independent Brownian bridges. In Section 3, we demonstrate the validity of the test throughout a simulation study. In Section 4, we provide the proofs of the results in Section 2. Concluding remarks are provided in Section 5.

2. CUSUM test for LM-FARIMA model

Let us consider the FARIMA model:

$$\Phi(B)(1 - B)^d y_t = \Psi(B)\epsilon_t, \tag{2.1}$$

where $\{\epsilon_t\}$ is a white noise process, B is the back-shift operator: $By_t = y_{t-1}$, $d \in (0, 1/2)$, $\Phi(B) = 1 - \sum_{i=1}^p \phi_i B^i$ and $\Psi(B) = 1 + \sum_{i=1}^q \psi_i B^i$. The fractional difference $(1 - B)^d$ is defined by the binomial series

$$(1 - B)^d = \sum_{j=0}^{\infty} \frac{\Gamma(j - d)}{\Gamma(j + 1)\Gamma(-d)} B^j,$$

where $\Gamma(\cdot)$ denotes the gamma function. Let $\theta = (d, \phi_1, \ldots, \phi_p, \psi_1, \ldots, \psi_q)'$ be the parameter vector of model (2.1). The parameter space Θ is a compact subset of $\mathcal{R}^{p+q+1}$ and for each $\theta \in \Theta$, the following conditions are satisfied:

(A1) Both $\Phi(z)$ and $\Psi(z)$ have roots outside the unit circle in the complex plane.
(A2) There exists a closed interval $[\underline{d}, \overline{d}] \subset (0, 1/2)$ such that $d \in [\underline{d}, \overline{d}]$.
(A3) $\phi_p \neq 0$, $\psi_q \neq 0$, and $\Phi(z)$ and $\Psi(z)$ have no common factors.

Suppose that given $y_1, \ldots, y_n$ from FARIMA(p, d, q) model in (2.1), one wishes to test

H_0: There are no changes in θ over the sample period vs.
H_1: There are abrupt changes in θ

based on the cusum test proposed by Lee et al. (2003). In order to construct the cusum test, we consider employing the quasi-log-likelihood estimator (QMLE) of θ as in Ling (2007). The QMLE is defined as the maximizer of the following objective function:

$$L_n(\theta) = -\frac{1}{2}\sum_{t=1}^{n} \epsilon_t^2(\theta), \tag{2.2}$$

where $\epsilon_t(\theta) = \Psi^{-1}(B)\Phi(B)(1 - B)^d y_t$. By (A1) and (A2), $\epsilon_t(\theta)$ can be expressed as

$$\epsilon_t(\theta) = \sum_{j=0}^{\infty} a_j(\theta) y_{t-j},$$

where $a_j(\theta)$ is the coefficient of z^j in the power series of $\Psi^{-1}(z)\Phi(z)(1 - z)^d$ and satisfies $a_j(\theta) = O(j^{-1-d})$ as $j \to \infty$. However, since only the n number of observations available, we use the modified objective function:

$$\tilde{L}_n(\theta) = -\frac{1}{2}\sum_{t=1}^{n} e_t^2(\theta) \quad \text{with } e_t(\theta) = \sum_{j=0}^{t-1} a_j(\theta) y_{t-j}$$

instead of the objective function L_n in (2.2), so that the estimator $\hat{\theta}_n$ is defined as a maximizer of $\tilde{L}_n(\theta)$ over Θ.