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Nonparametric tests for detecting greater residual life times

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Abstract

A nonparametric procedure is proposed for testing the exponentiality against the hypothesis that one life distribution has a greater residual life time than the other life distribution. Such a hypothesis turns out to be equivalent to the one that one failure rate is greater than the other and so the proposed test works as a competitor to more IFR tests given by Kochar [Kochar, S. C. (1979). Distribution-free comparison of two probability distributions with reference to their hazard rate. *Biometrika, 66*, 437–441; Kochar, S. C. (1981). A new distribution-free test for the equality of two failure rates. *Biometrika, 68*, 423–426] and Cheng [Cheng, K. F. (1985). Tests for the equality of failure rates. *Biometrika, 72*, 211–215]. Our test statistic utilizes the U-statistics theory and a large sample nonparametric test is established. The power of the proposed test is discussed by calculating the Pitman asymptotic relative efficiencies against several alternative hypotheses. A numerical example is presented to exemplify the proposed test.

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1. Introduction

In reliability theory the concept of ageing plays a fundamental role in classifying the life distributions. The classes of increasing failure rate (IFR) and increasing failure rate average (IFRA) are based on the monotonicity pattern of failure rate of the distribution and the classes of decreasing mean residual life (DMRL) and new better than used in expectation (NBUE) are classified by the pattern of the mean residual life. The class of new better than used (NBU) is defined by utilizing the stochastic ordering of the residual life time (RLT). The dual classes of DFR, DFRA, IMRL, NWUE and NWU are defined by reversing the pattern of failure rate, mean residual life and residual life time. The border distribution for all of the above classes is the exponential distribution. To test the null hypothesis that the distribution is exponential against the alternative that the distribution belongs to one of the above classes, many authors have proposed a number of nonparametric tests in the literature. Ahmad (1975, 2001), Barlow and Proschan (1969) and Proschan and Pyke (1967) propose the IFR tests. For the NBU and NBUE alternatives, there exist the nonparametric tests by Ahmad (2001), Hollander and Proschan (1972), Hollander and Proschan (1975) and Koul

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(1977) among many others. Aly (1990) and Hollander and Proschan (1975) also propose a class of tests against the DMRL alternatives.

Since Chikkagoudar and Schuster (1974) considered the problem of comparing two populations in terms of their failure rates, Cheng (1985) and Kochar (1979, 1981) have developed several nonparametric procedures for testing the null hypothesis that two life distributions are equal against the alternative that one failure rate dominates the other. Such tests are proved to be useful for comparing two used items with different life distributions in terms of their degradation processes as they age. There exist other tests for comparing two life distributions with respect to their NBU-ness or IFRA-ness by Hollander, Park, and Proschan (1986), Lim, Kim, and Park (2004, 2005) and Tiwari and Zalkikar (1991).

Let $X \geq 0$ be a life time random variable with continuous survival function $\bar{F}(x) = P(X > x)$. Then the residual life time at age t, denoted by X_t, is a random variable with continuous survival function $\bar{F}_t(x) = \bar{F}(x+t)/\bar{F}(t),\ x, t \geq 0$. In this paper we propose a new nonparametric procedure for testing the equality of two life distributions against the alternative that one life distribution has a greater residual life time than the other life distribution. The following situation illustrates how our proposed test might prove useful. Suppose that there are two groups of patients suffering from a certain type of cancer, one group being treated by a certain medical treatment and the other group being a placebo group. To check whether the medical treatment is effective in treating that particular type of cancer, the medical authority wishes to test the hypothesis that the residual life time of the treatment group is stochastically greater than the residual life time of the placebo group after the treatment group receives the treatment for a certain length of time, while the placebo group is not treated at all for the same length of time. If such a hypothesis is accepted, then the treatment is verified to be effective and the treatment group may have a longer residual life than the placebo group. To the best of our knowledge, no other tests have been proposed for testing the stochastic ordering of two residual life times. However, it can be shown that the stochastic ordering of residual life times is equivalent to the failure rate ordering. Thus, the efficiency of our proposed test can be evaluated by calculating the Pitman asymptotic relative efficiencies with respect to the more IFR tests by Cheng (1985) and Kochar (1979, 1981).

In Section 2, we derive the test statistics for detecting greater residual life time property. Section 3 proposes a nonparametric test procedure by proving asymptotic normality of the proposed test statistic. The consistency of the test is also proved. Sections 4 and 5 present the Pitman asymptotic relative efficiency results and a numerical example.

2. Test statistic for residual life time

Let X and Y be random nonnegative life times of two systems with continuous distribution functions $F(t)$ and $G(t)$ and let X_t and Y_t be the corresponding residual life times at age t, respectively. In this section we develop a test statistic for testing the null hypothesis

$H_0 : F = G$ (the common distribution is unspecified)

versus the alternative hypothesis

$H_a : X_t \leq Y_t$ for all $t \geq 0$, with strict inequality holding for some t,

based on two random samples $X_1, \ldots, X_m$ and $Y_1, \ldots, Y_n$ taken from F and G, respectively. We assume that $\underline{X} = (X_1, \ldots, X_m)$ and $\underline{Y} = (Y_1, \ldots, Y_n)$ are independent. H_a states that the residual life time at age t is stochastically greater when the underlying distribution is F than when the underlying distribution is G. Note that under H_a, $\bar{F}(x+t)\bar{G}(t) \leq \bar{G}(x+t)\bar{F}(t)$ for all $t \geq 0$. Our test statistic is derived based on the following parameter:

$$\begin{aligned}\Delta(F, G) &\overset{def}{=} 2\int_0^\infty \int_0^\infty \{\bar{G}(x+t)\bar{F}(t) - \bar{F}(x+t)\bar{G}(t)\}\mathrm{d}G(x+t)\mathrm{d}F(t) \\ &= \int_0^\infty \bar{G}^2(t)\bar{F}(t)\mathrm{d}F(t) - 2\int_0^\infty \int_t^\infty \bar{F}(u)\bar{G}(t)\mathrm{d}G(u)\mathrm{d}F(t).\end{aligned}$$

Under H_0, $\Delta(F, G) = 0$ and under H_a, $\Delta(F, G) > 0$. Thus $\Delta(F, G)$ can be used as a measure of departure from H_0 and the larger value of $\Delta(F, G)$ indicates that Y_t is stochastically greater than X_t. Let $F_m(\bar{F}_m)$ and $G_n(\bar{G}_n)$ denote the empirical (survival) functions formed by random samples $\underline{X}$ and $\underline{Y}$, respectively. A natural nonparametric test statistic for testing H_0 versus H_a can be formed by substituting F_m and G_n in place of F and G of $\Delta(F, G)$, respectively, as