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Plug-in prediction intervals for a special class of standard ARH(1) processes

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ABSTRACT

This paper studies the asymptotic properties of a plug-in predictor, based on the formulation of a componentwise estimator of the autocorrelation operator, for a special class of standard autoregressive Hilbertian processes of order one (ARH(1) processes). In the Gaussian case, double asymptotic functional plug-in prediction intervals are derived. Some numerical examples are considered for illustration.

1. Introduction

In the last few decades, a vast literature has been developed in relation to functional data analysis techniques. We highlight the nice summary on the statistics theory with functional data in [10], and the references therein. Recent developments can also be found in [18], where inference based on second order statistics, and functional principal component analysis is provided. In this book, both, inference for independent and dependent functional data structures are covered. New branches of univariate and multivariate functional statistics are also introduced in [6].

Functional prediction in the framework of Autoregressive Hilbertian processes of order one (ARH(1) processes) has been initially addressed in [7,8]. This prediction problem requires the estimation of the autocorrelation operator of an ARH(1) process. The inverse problem associated with the estimation of the autocorrelation operator by the method of moments has been extensively studied (see, for example, [4,12,21,23,24,26]). In particular, Antoniadis and Sapatinas [2] adopt deterministic approaches, from the theory of function estimation in linear ill-posed inverse problems, but these approaches are not computationally efficient for large sample sizes. They then propose three linear wavelet methods to get efficiency. Specifically, regularization of the sample paths of the stochastic process is achieved, and consistency of the corresponding

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prediction estimators is obtained. This approach can be applied to data collected as curves indexed by time intervals of equal lengths. These intervals can be adjacent, disjoint or overlapping (see, for example, [27,28]). Until our knowledge, ARH(1) prediction, based on confidence intervals, has not been addressed yet. This fact motivates the subject of the present paper.

On the other hand, in the nonparametric functional statistical framework, we can refer the reader to the monograph by Ferraty and Vieu [16]. Ferraty and Vieu [15] establish the convergence rates for kernel estimators. Asymptotic normality was extended to the functional context in [14,22] for independent and identically distributed, and strongly mixing sequence, respectively. Kudraszow and Vieu [19] study nearest neighbor regression. Aneiros-Perez and Vieu [1] consider the semiparametric functional framework in the context of partially linear models, where the nonparametric part is estimated by the Nadaraya–Watson estimator. Asymptotically valid point-wise confidence intervals can be constructed from the results by Ferraty et al. [14], where asymptotic normality of the Nadaraya–Watson estimator is proved with explicit expressions for the bias and variance terms. The expressions for bias and variance involve unknown parameters, and must be estimated from the data which hinders their application. Lian [20] proposes to adapt the empirical likelihood method (see, for example, [25]), to construct point-wise confidence intervals for the regression function.

In this paper, a special class of standard ARH(1) processes is considered, whose autocorrelation operator is a bounded linear operator with norm less than one. This operator admits a diagonal spectral representation with respect to the eigenvectors of the auto-covariance operator of the ARH(1) process considered. Its pure point spectrum displays an accumulation point at one, although the modulus of all the eigenvalues of the autocorrelation operator are less than one. Under the setting of conditions assumed, this class of autocorrelation operators naturally arises when the decay velocity of the eigenvalues of the covariance operator of the innovation process is faster than the one displayed by the eigenvalues of the covariance operator of the autoregressive process. A componentwise estimator of the autocorrelation operator, as well as its associated plug-in predictor are formulated. Their asymptotic properties are studied. Double asymptotic projection plug-in confidence intervals are also established, to approximate the future functional values of this analyzed class of standard ARH(1) process, in the Gaussian case.

The outline of the paper is as follows: Preliminary elements are introduced in Section 2. In particular, the setting of conditions required for the definition of the class of standard ARH(1) processes studied are also given in this section. Asymptotic results and double asymptotic confidence intervals are derived in Section 3. In Section 4, some numerical examples are considered to illustrate the obtained results. Final comments are provided in Section 5.

2. A special class of standard ARH(1) processes

Let us consider the definition of standard ARH(1) processes (see, for example, [8]).

Definition 1. Let H be a separable real-valued Hilbert space. A sequence $Y = (Y_t,\ t \in \mathbb{Z})$ of H-valued random variables on a basic probability space $(\Omega, \mathcal{A}, P)$ is called an autoregressive Hilbertian process of order 1, associated with (μ, ε, ρ), if it is stationary and satisfies

$$X_t = Y_t - \mu = \rho(Y_{t-1} - \mu) + \varepsilon_t = \rho(X_{t-1}) + \varepsilon_t, \quad t \in \mathbb{Z}, \tag{1}$$

where $\varepsilon = (\varepsilon_t,\ t \in \mathbb{Z})$ is a Hilbert-valued white noise in the strong sense (i.e., a zero-mean stationary sequence of independent H-valued random variables with $E\|\varepsilon_t\|_H^2 = \sigma^2 < \infty$, for every $t > 0$), and $\rho \in \mathcal{L}(H)$, with $\mathcal{L}(H)$ being the space of linear bounded operators on H. For each $t > 0$, ε_t and X_{t-1} are assumed to be uncorrelated.

If there exists a positive $j_0 \geq 1$ such that $\|\rho^{j_0}\|_{\mathcal{L}(H)} < 1$, then, the ARH(1) process X in (1) is standard, and there exists a unique stationary solution to Eq. (1) (see, for example, [8, Chapter 3]).

The auto-covariance and cross-covariance operators are given by

$$\begin{aligned} C_X &= E[X_t \otimes X_t] = E[X_0 \otimes X_0], \quad t \in \mathbb{Z}, \\ D_X &= E[X_t \otimes X_{t+1}] = E[X_0 \otimes X_1], \quad t \in \mathbb{Z}, \end{aligned} \tag{2}$$

where, for $f, g \in H$,

$$f \otimes g(h) = f\, \langle g, h \rangle_H$$

defines a Hilbert–Schmidt operator on H. Operator C_X is in the trace class. In particular, $E\|X_t\|_H^2 < \infty$, for all $t \in \mathbb{Z}$.

From Eqs. (1) and (2), for all $h \in H$,

$$\begin{aligned} D_X(h) &= \langle D_X, h \rangle_H = \langle E[X_t \otimes X_{t+1}], h \rangle_H \\ &= E\left[X_t \,\langle \rho(X_t), h \rangle_H\right] + \langle E\left[X_t \otimes \varepsilon_{t+1}\right], h \rangle_H \\ &= E\left[X_t \left\langle X_t, \rho^*(h) \right\rangle_H\right] + 0 \\ &= \left\langle E\left[X_t \otimes X_t\right], \rho^*(h) \right\rangle_H \\ &= \left\langle C_X, \rho^*(h) \right\rangle_H = \langle \rho C_X, h \rangle_H = \rho C_X(h), \end{aligned} \tag{3}$$

where we have interchanged the expectation with the inner product in H, since $X_t \in \mathcal{L}_H^2(\Omega, \mathcal{A}, P)$, for all $t \in \mathbb{Z}$.