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Multivariate trend function testing with mixed stationary and integrated disturbances☆

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ABSTRACT

Trend models are important in describing nonstationary behavior of a time series. In this paper we propose valid tests for the trend coefficients in a multivariate system with mixed stationary, integrated or nearly integrated errors. Cross-sectional and serial dependence in innovations are left unspecified beyond regularity assumptions. We consider two sets of tests based on OLS and SUR estimation of the transformed system. A modified SUR estimator corrected for serial correlation of unknown form is shown to be asymptotically efficient. The standard tests under stationarity are also analyzed and potential misleading inferences are demonstrated. The framework is general allowing for linear and nonlinear trend functions. Asymptotic theory, simulations and an empirical application are provided.

1. Introduction

Deterministic trend model is a simple and effective way to mimic the nonstationary behavior in the mean of an economic time series. In its simplest form, the trend is modeled as a linear function of time, and the slope coefficient has interpretation of average growth or growth rate. A broken trend or piecewise linear trend model may be entertained if the growth rate is changed after some time point. For example, Estrada, Perron and Martínez-López [13] noticed the slow-down in warming of the global climate system and analyzed its possible cause by human influences using a broken trend model. On the other hand, it is well known that correctly modeling the time trend is of crucial importance in testing of a unit root and cointegration; see [32,28] for surveys on these topics. Recent research on trend models concerns estimation and inference that are robust to weak and strong serial correlation of disturbances; see [6,39,22,35,3,18,4,29,43] for the linear trend model, and Sayginsoy and Vogelsang [36] and Harvey, Leybourne and Taylor [19] for the broken trend model.

While univariate trend models have received considerable attention, such as the references cited above, there is relatively much less research on multivariate trend function modeling. Given the interactive nature of most economic time series, reliable inferential approaches in the multivariate context are generally preferable to those built on modeling them individually. Moreover, joint inference of trend parameters for cross-sectional units provides meaningful interpretation

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of economic hypotheses that cannot be offered by univariate models. Vogelsang and Franses [40] considered inference of a multivariate linear trend model and presented an alternative approach to study economic convergence by testing whether cross-country observations of a macroeconomic time series share a common deterministic trend. Under the same model as Vogelsang and Franses, Xu [41] studied the impact of the nonstationary volatility process on the multivariate trend tests and proposed robust alternatives, while Sun [38] focused on testing-optimal selection of the truncation parameter in the long run variance estimator. These papers all assume the disturbances follow a multivariate stable and weakly dependent process. There is a growing recent literature in which the deterministic trend is modeled as an unknown smooth nonparametric function [5,24]. In the multivariate case, Atak, Linton and Xiao [2] proposed a partially linear model to study climate change in UK assuming a common trend function across stations while allowing heterogeneous coefficients for other covariates. In a similar partially linear model, Zhang, Su and Phillips [45] proposed a nonparametric test for a common trend allowing for fixed effects.[1] Stationarity and weak dependence are also assumed in these papers.[2]

In the current paper, we propose tests for the multivariate trend function that allow some disturbances in the system to be integrated or nearly integrated. The issue is of empirical relevance since it has been long recognized since at least Nelson and Plosser [26] that many macroeconomic time series are characterized as a nonstationary process that has no tendency to return to a deterministic path, i.e. a (nearly) I(1) process around a deterministic mean. In theory, inference of the coefficients in the regression with integrated disturbances is known to be nonstandard and requires a different asymptotic theory from the one with stationary disturbances.[3] In our setting, when some disturbance component in the multivariate time series under scrutiny is nonstationary, the extant trend tests mentioned above fail. Different convergence rates of trend coefficients across equations present new challenges for inference. In general, we are mainly interested in testing for the dominant regression coefficients (i.e. trend coefficients) across equations allowing for different orders of integration in disturbances. Our framework allows for linear and nonlinear trends.

The paper is organized as follows. The framework and assumptions are described in Section 2, and models nested in the framework are exemplified. Applying a transformation to the original system, tests based on equation-by-equation and cross-equation estimation of the trend coefficients are proposed in Sections 3 and 4 respectively, and limit theories are provided. We also show that efficient estimation can be achieved by a modified SUR estimator with correction for serial correlation when there are at least one component with I(0) disturbances and one with I(1) disturbances. Standard tests treating all disturbances as stationary are examined in Section 5. Section 6 presents simulations and a simple application to illustrate the small-sample behaviors of the tests studied. Extensions are discussed in Section 7. Section 8 concludes. Technical proofs are collected in Appendices. For empirical researchers, the implementation codes in MATLAB (for the proposed inference methods in Sections 3 and 4), together with the data used in Section 6 (for replication purpose), are available on the author's website.

2. The framework and the assumptions

Consider an m-dimensional $(m \times 1)$ time series y_t $(t = 0, 1, \ldots, T)$ which follows

$$y_t = \alpha' f_0(t) + \beta f(t) + u_t, \tag{1}$$

where $f_0(t)$ and $f(t)$ are $k \times 1$ and scalar, respectively, deterministic functions, and α and β are $k \times m$ and $m \times 1$ unknown parameters. The first element of $f_0(t)$ is usually a constant. The disturbance term u_t accounts for serial dependence in the data which is left unspecified. The system (1) is essentially a multivariate regression model with the same regressors for all equations. The formulation of the two parts of regressors in (1) is such that the main interest is in cross-sectional simultaneous inference of the coefficient β.

The two leading cases are the linear trend model (Case I) and the broken trend model (Case II) in which the forms of $f_0(t)$ and $f(t)$ are specified in Table 1. Case II is also nested in the framework of (1) if it is extended to allow for a simultaneous break in the intercept. The hypothesis of interest can be, e.g. whether β shares the same value across equations, which in the two cases corresponds to the conjectures of a common (deterministic) trend and a common break at some given time point. In our theoretical development we allow $f_0(t)$ and $f(t)$ to assume general functional forms as long as the following conditions are satisfied.

Assumption 1. (i) There exist a non-degenerate function $F(r)$ and a non-negative real number $\delta > -1/2$ such that $f(t)/T^\delta = f(\lfloor Tr \rfloor)/T^\delta \to F(r)$ as $T \to \infty$, where $\lfloor \cdot \rfloor$ means the integer part. There exists a diagonal matrix D_T such that

[1] Other recent papers on testing coefficient heterogeneity of non-trending regressors across equations are also related. Ray and Savin [33] considered testing a common zero intercept in a multi-equation Fama–French three-factor model of portfolio returns to check model adequacy. Pesaran and Yamagata [30] is concerned about testing for slope heterogeneity in a panel data model when the number of equations is large.

[2] For more applications of multivariate trend testing, in a study of the dynamics of gains from sectoral and geographic diversification and the link to the real economy, Carrieri et al. [7] found comparable increase in conditional correlation in equity returns and industrial production among developed markets but not the emerging markets, which has implications for asset allocation. Focusing on emerging equity market comovements, Eiling and Gerard [11] found significant positive time trends in cross-country correlations within geographical regions and correlations across regions, but also showed important heterogeneity across regions in the speed at which comovements increase over time, which might be explained by different roles of equity market openness and development together with trade openness played in different continents.

[3] See [31] for the treatment of a general regression model (spurious regression or non-cointegrated regression).