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Bayesian analysis of multivariate stable distributions using one-dimensional projections

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ABSTRACT

In this paper we take up Bayesian inference in general multivariate stable distributions. We exploit the representation of Matsui and Takemura (2009) for univariate projections, and the representation of the distributions in terms of their spectral measure. We present efficient MCMC schemes to perform the computations when the spectral measure is approximated discretely or, as we propose, by a normal distribution. Appropriate latent variables are introduced to implement MCMC. In relation to the discrete approximation, we propose efficient computational schemes based on the characteristic function.

1. Introduction

Univariate stable distributions have been thoroughly studied in econometrics, statistics and finance over the past few decades [31]. Their empirical application is still hampered by the fact that their density is not available in analytic form, despite advances in Bayesian computation using MCMC. Buckle [3] and Tsionas [32] provided Gibbs sampling schemes for general and symmetric stable distributions, respectively. The problem is that the conditional posterior distributions of certain latent variables are cumbersome to work with and require careful tuning. The analogous problem in the multivariate case is exceedingly difficult although a few attempts have been made to solve it. The impediment is that multivariate stable distributions, unlike the univariate case, are defined through their spectral measures which, in practice, are unknown. Ravishanker and Qiou [30] for example, proposed an EM algorithm based on Buckle [3] in the case of symmetric isotropic stable distributions but this class is too narrow to be of empirical importance. It is defined by the transformation $X = \mu + C\xi$, where ξ is a vector of independent random variables each one distributed as standard symmetric stable, μ is a vector of location parameters, Σ is a scale matrix, and $C^\top C = \Sigma$. It is known that the class of elliptical stable distributions can be defined through the transformation $X = \mu + RCu$ where u is uniformly distributed on the unit sphere $\mathbb{S}^{d-1} = \{\boldsymbol{x} \in \mathbb{R}^d | \|\boldsymbol{x}\| = 1\}$, C is a $d \times d$ scale matrix of full rank, and $R = \sqrt{VS_{\alpha/2}}$ where, independently, $V \sim \chi^2_d$ and $S_{\alpha/2}$ follows a stable distribution with parameter $\alpha/2$ and maximal skewness $\beta = 1$. Of course not all multivariate stable distributions are elliptical. See [14]. When $V \sim \chi^2_1$ the distribution of X is in the class of elliptically contoured stable distributions [24, p. 2].

In connection with multivariate stable Paretian distributions, even the computation of the characteristic functions becomes complicated because they are only defined through their spectral measure, an object that is needed in order to retain the equivalence between the density and the characteristic function. The estimation of the spectral measure itself has

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proved itself to be quite cumbersome even for bivariate distributions (see the seminal works of McCulloch [17,19], Nolan et al. [25], and Nolan and Rajput [26]).

The present paper is related to recent advances in the econometrics of stable distributions. Dominicy and Veredas [8] propose a method of quantiles to fit symmetric stable distributions. Since the quantiles are not available in closed form they are obtained using simulation resulting in the method of simulated quantiles or MSQ. Hallin et al. [10] propose an easy-to-implement R-estimation procedure which remains consistent contrary to least squares with stable disturbances. Broda et al. [2] propose a new stable mixture GARCH model that encompasses several alternatives and can be extended easily to the multivariate asset returns case using independent components analysis. Ogata [27] uses a discrete approximation to the spectral measure of multivariate stable distributions and proposes estimating the parameters by equating the theoretical and empirical characteristic function in a generalized empirical likelihood/GMM framework.

Relative to this work, we show how to implement Bayesian inference for multivariate stable distributions by providing statistical inferences about the spectral measure jointly with the other parameters of the model. For numerical analysis via MCMC we employ a novel data augmentation technique for stable distributions. We use a discrete approximation of the measure where the configuration and the number of points are unknown. We also propose a novel approximation to the spectral measure based on a multivariate normal distribution.

2. Stable distributions

A random variable X is called strictly (univariate) stable if for all n, $\sum_{i=1}^{n} X_i \sim c_n X$ for some constant c_n, where $X_1, \ldots, X_n$ are independently distributed with the same distribution as X. It is known that the only possible choice is to have $c_n = n^{1/\alpha}$ for some $\alpha \in (0, 2]$. General non-symmetric stable distributions are defined via the log characteristic function which is given by the following expression [31,36]:

$$\begin{aligned}\log \varphi(\tau) &= \log E \exp(\iota \tau X) \\ &= \begin{cases} \iota\mu\tau - |\sigma\tau|^{\alpha}\left\{1 - \iota\beta \operatorname{sgn}(\tau)\tan\frac{\pi\alpha}{2}\right\}, & \alpha \neq 1 \\ \iota\mu\tau - \sigma|\tau|\left\{1 + \iota\beta \operatorname{sgn}(\tau)\frac{2}{\pi}\log|\tau|\right\}, & \alpha = 1, \end{cases}\end{aligned} \tag{1}$$

where $\tau \in \mathbb{R}$, μ and σ are location and scale parameters, α is the characteristic exponent, $\beta \in [-1, 1]$ is the skewness parameter, and $\iota = \sqrt{-1}$. In this paper we are interested in multivariate stable distributions, that is distributions of a random variable in $\mathbb{R}^d$. Suppose X is a vector of random variables with characteristic exponent $\alpha \in (0, 2]$. Its characteristic function is $\varphi_X(\boldsymbol{\tau}) = E\exp\{\iota\langle\boldsymbol{\tau}, X\rangle) = \exp\left(-I_X(\boldsymbol{\tau}) + \iota\langle\tau, \mu\rangle\right)$ where $\langle\boldsymbol{\tau}, X\rangle = \boldsymbol{\tau}^{\top}X$ denotes inner product, and

$$I_X(\boldsymbol{\tau}) = \int_{\mathbb{S}^{d-1}} \psi_{\alpha}\left(\langle\boldsymbol{\tau}, \boldsymbol{s}\rangle\right) \Gamma(d\boldsymbol{s}), \tag{2}$$

where $\mathbb{S}^{d-1} = \left\{\boldsymbol{u} \in \mathbb{R}^d | \langle\boldsymbol{u}, \boldsymbol{u}\rangle = 1\right\}$ is the boundary of the unit ball in $\mathbb{R}^d$, Γ is a finite Borel measure of the vector X, called the spectral measure, $\mu \in \mathbb{R}^d$ is a vector of location parameters, and the complex function ψ is defined as follows:

$$\psi_{\alpha}(u) = \begin{cases} |u|^{\alpha}\left\{1 - \iota \operatorname{sgn}(u)\tan\frac{\pi\alpha}{2}\right\}, & \alpha \neq 1, \\ |u|\left\{1 + \iota\frac{2}{\pi}\operatorname{sgn}(u)\log|u|\right\}, & \alpha = 1. \end{cases} \tag{3}$$

See seminal work by Nolan [24], Nolan and Rajput [26], Abdul-Hamid and Nolan [1], and also Cambanis and Miller [5], and Nagaev [22]. Notably the parameters (α, Γ) fully define all centered multivariate stable distributions, and a skewness parameter β is not needed[1] in this case, since we have the full measure, Γ. We denote the class by $X \sim \mathscr{S}_{\alpha,d}(\mu, \Gamma)$. Press [29] attempted to define a multivariate α-stable distribution without using the spectral measure Γ. Later on Paulauskas [28] provided some corrections as not all α-stable distributions can be represented using Press' [29] characteristic function. Cheng and Rachev [6] is an interesting paper where the authors provided estimates of the spectral measure as well as applications to a stable portfolio. It is notable that the projection of X on τ, viz. $\langle\boldsymbol{\tau}, X\rangle$ has a univariate stable distribution. The characteristic exponent α remains the same but scale, location and skewness depend on $\boldsymbol{\tau}$. The multivariate characteristic function is not easy to work with as in the univariate case because of the dependence on the spectral measure. As this can rarely be specified in advance, it is necessary to provide posterior inferences about it, in the context of Bayesian analysis.

One approach [4] is to assume that Γ can be approximated by a discrete measure, in which case we have:

$$\Gamma(d\boldsymbol{s}) = \sum_{j=1}^{J} \gamma_j \delta_{\{\boldsymbol{s}^{(j)}\}}(d\boldsymbol{s}), \tag{4}$$

[1] Actually, there are skewness parameters $\beta(\boldsymbol{\tau})$ that depend on the particular projection $\boldsymbol{\tau}$.