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Multivariate families of gamma-generated distributions with finite or infinite support above or below the diagonal

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ABSTRACT

In this paper, we introduce two new families of multivariate distributions with finite or infinite support above or below the diagonal generated by McKay's bivariate gamma distribution and show that their conditional distributions are univariate gamma- and beta-generated distributions. We derive the Shannon entropies of the introduced families of bivariate distributions. We then focus on the special cases of bivariate gamma-exponentiated exponential distributions, and discuss their properties. Finally, we illustrate the usefulness of the proposed bivariate gamma-exponentiated exponential distributions with a real dataset.

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1. Introduction

We introduce here two families of multivariate distributions with finite or infinite support on $a < x < y < b$ (support above the diagonal) and $a < y < x < b$ (support below the diagonal), respectively, where a and b can be finite or infinite numbers. Let F be the cumulative distribution function (cdf), f be the corresponding probability density function (pdf), and $\overline{F}$ be the survival function given by $\overline{F}(x) = 1 - F(x)$. The first family of multivariate distributions is given by the joint density

$$g(\mathbf{x}) = \left(\prod_{i=1}^{r} \Gamma(\delta_i)\right)^{-1} (-\log \overline{F}(x_1))^{\delta_1 - 1} \prod_{i=1}^{r-1} \left(-\log \frac{\overline{F}(x_{i+1})}{\overline{F}(x_i)}\right)^{\delta_{i+1}-1} \prod_{i=1}^{r-1} \frac{f(x_i)}{\overline{F}(x_i)} f(x_r), \tag{1}$$

where $r \geq 1$, $\mathbf{x} = (x_1, x_2, \ldots, x_r)$, $a < x_1 < \cdots < x_r < b$, and $\delta_i > 0$ for $i = 1, 2, \ldots, r$. The second family of multivariate distributions is given by the joint density

$$g(\mathbf{x}) = \left(\prod_{i=1}^{r} \Gamma(\delta_i)\right)^{-1} (-\log F(x_1))^{\delta_1 - 1} \prod_{i=1}^{r-1} \left(-\log \frac{F(x_{i+1})}{F(x_i)}\right)^{\delta_{i+1}-1} \prod_{i=1}^{r-1} \frac{f(x_i)}{F(x_i)} f(x_r), \tag{2}$$

where $r \geq 1$, $\mathbf{x} = (x_1, x_2, \ldots, x_r)$, $a < x_r < \cdots < x_1 < b$, and $\delta_i > 0$ for all $i \in \{1, 2, \ldots, r\}$. These families of distributions have two or more additional parameters that control shapes and skewness properties of the introduced distributions.

These families of distributions can be motivated as follows. First motivation is similar to the motivation of Jones and Larsen [11] who considered a possible application of the multivariate families of distributions defined above the diagonal.

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In the case of bivariate distributions defined above the diagonal, Jones and Larsen [11] have noted that one natural scenario to obtain $X_i < Y_i$, for all $i = 1, 2, \ldots, n$, is to consider X as a minimum and Y as a maximum of some quantity. They refer to [10] in which an example with minimum and maximum temperatures is given.

Second motivation follows from the joint distribution of the upper and lower record values, respectively. Let $X_{U(1)}, \ldots, X_{U(n)}, \ldots$, and $X_{L(1)}, \ldots, X_{L(n)}, \ldots$, be the upper and lower record values, respectively, from a sequence of i.i.d. continuous random variables with parent distribution F. Then, the joint pdf of the upper record values $X_{U(m_1)}, \ldots, X_{U(m_r)}$ is given by (1) with $\delta_1 = m_1$, $\delta_i = m_i - m_{i-1}$, for $i = 2, 3, \ldots, r$, and $m_i < m_{i+1}$ for $i = 1, 2, \ldots, r-1$. Similarly, the joint pdf of the lower record values $X_{L(m_1)}, \ldots, X_{L(m_r)}$ is given by (2) with $\delta_1 = m_1$, $\delta_i = m_i - m_{i-1}$, for $i = 2, 3, \ldots, r$, and $m_i < m_{i+1}$ for $i = 1, 2, \ldots, r-1$.

Some other multivariate and bivariate distributions with support above the diagonal have also been studied in the past. McKay [15] introduced a bivariate gamma distribution, while Mihram and Hultquist [16] generalized this distribution and introduced the Beta-Stacy distribution. Two more forms of multivariate gamma distributions have been introduced by Mathai and Moschopoulos [14] and Furman [7]. Jones and Larsen [11] introduced a family of multivariate distributions and used it for modelling ordered multivariate data.

The rest of this paper is organized as follows. In Section 2, we provide some connections between the introduced families of bivariate distributions and McKay's bivariate gamma distribution. The conditional pdfs of X, given $Y = y$, and of Y, given $X = x$, are derived. The Shannon entropies are derived in Section 3. In Section 4, the special case of bivariate gamma-exponentiated exponential distribution is considered and its properties are discussed. In Section 5, we discuss the estimation of the model parameters of bivariate gamma-exponentiated exponential distribution through maximum likelihood method. An illustration with a real dataset is finally made in Section 6.

2. Bivariate families and their properties

Let us now consider the bivariate case of the families given in (1) and (2). First bivariate family of distributions is given by the joint pdf of the form

$$g_{X,Y}(x, y) = \frac{1}{\Gamma(\delta)\Gamma(\nu)} \left(-\log \overline{F}(x)\right)^{\delta-1} \left(-\log \frac{\overline{F}(y)}{\overline{F}(x)}\right)^{\nu-1} \frac{f(x)f(y)}{\overline{F}(x)}, \tag{3}$$

where $a < x < y < b$, $\delta > 0$ and $\nu > 0$. Direct integration yields the marginal pdf of the random variable X as

$$g_X(x) = \frac{1}{\Gamma(\delta)} \left(-\log \overline{F}(x)\right)^{\delta-1} f(x), \quad a < x < b, \tag{4}$$

which shows that the marginal distribution of the random variable X belongs to the family of gamma-generated random variables with parameter δ, introduced by Zografos and Balakrishnan [21]. These authors showed that the cdf of the random variable X is given by

$$G(x) = \frac{1}{\Gamma(\delta)} \int_0^{-\log \overline{F}(x)} t^{\delta-1} e^{-t} dt, \quad a < x < b,$$

justifying the name gamma-generated family. Similarly, we can show that the marginal pdf of the random variable Y is given by

$$g_Y(y) = \frac{1}{\Gamma(\delta+\nu)} \left(-\log \overline{F}(y)\right)^{\delta+\nu-1} f(y), \quad a < y < b,$$

which implies that the marginal distribution of the random variable Y also belongs to the family of gamma-generated random variables with parameter $\delta + \nu$.

In an analogous manner, we can consider the other bivariate family of distributions with joint pdf of the form

$$g_{X,Y}(x, y) = \frac{1}{\Gamma(\delta)\Gamma(\nu)} \left(-\log F(x)\right)^{\delta-1} \left(-\log \frac{F(y)}{F(x)}\right)^{\nu-1} \frac{f(x)f(y)}{F(x)}, \tag{5}$$

where $a < y < x < b$, $\delta > 0$ and $\nu > 0$. We can show that the corresponding marginal pdfs of random variables X and Y are, respectively, given by

$$g_X(x) = \frac{1}{\Gamma(\delta)} \left(-\log F(x)\right)^{\delta-1} f(x), \quad a < x < b,$$

$$g_Y(y) = \frac{1}{\Gamma(\delta+\nu)} \left(-\log F(y)\right)^{\delta+\nu-1} f(y), \quad a < y < b.$$

In this case, we observe that the marginal distributions of the random variables X and Y belong to the family of univariate distributions introduced by Ristić and Balakrishnan [18].

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