Journal of Multivariate Analysis 143 (2016) 275–298

Contents lists available at ScienceDirect

Journal of Multivariate Analysis

journal homepage: www.elsevier.com/locate/jmva

Stochastic dominance and statistical preference for random variables coupled by an Archimedean copula or by the Fréchet–Hoeffding upper bound

Ignacio Montes, Susana Montes *

Department of Statistics and O.R., University of Oviedo, Spain

ARTICLE INFO

Article history:
Received 28 November 2014
Available online 1 October 2015

AMS subject classifications:
60E15
62H30

Keywords:
Stochastic dominance
Statistical preference
Archimedean copula
Comonotone random variables
Countermonotone random variables

ABSTRACT

Stochastic dominance and statistical preference are stochastic orders with different interpretations: the former is based on the comparison of the marginal distributions while the latter is based on the joint distribution. Sklar's Theorem allows expressing the joint distribution in terms of the marginals by means of a copula. This paper investigates the relationship between these two stochastic orders for comonotone random variables and random variables coupled by an Archimedean copula.

1. Introduction

In many real life situations we usually have to choose among different alternatives. When the alternatives are defined under randomness, they are usually modeled by means of random variables. The comparison of random variables is a longstanding problem that has been tackled from many points of view (see among others [2,17,26,27]).

Stochastic orders [24,26] are methods that allow the comparison of uncertain quantities. This theory is a very popular topic within Economics [18], Finance [19] or Social Welfare [15], among others. One of the most usual ways of ordering random variables is stochastic dominance [18]. First degree stochastic dominance, that seems to be the most widely used method, orders random variables by comparing their cumulative distribution functions. Its main drawback is that it imposes a very strong condition to get an order, so many pairs of random variables are deemed incomparable. Because of this fact, the condition it imposes can be soften, and it gives rise to the nth degree stochastic dominance.

The other stochastic order we shall use is statistical preference [11,13]. It is based on a probabilistic relation [3] that is defined from the joint distribution of the variables, and therefore it uses all the information about them. Furthermore, this method provides degrees of preference to measure the strength of the preference of one variable over the other and it compares any pair of random variables. According to [14], this is the optimal method to compare qualitative variables. The main drawback of this stochastic order is its lack of transitivity [9,8,10,12,21].

We have already said that statistical preference is based on the joint distribution. The well-known Sklar's Theorem allows to express the joint distribution in terms of the marginals by means of a function called copula [25]. Some usual instances

* Corresponding author.
E-mail addresses: imontes@uniovi.es (I. Montes), montes@uniovi.es (S. Montes).

http://dx.doi.org/10.1016/j.jmva.2015.09.015
0047-259X/

of copulas are the product copula and the Fréchet–Hoeffding upper and lower bounds, that give rise to independent, comonotone and countermonotone random variables, respectively. The product copula and the lower Fréchet–Hoeffding bounds are included in a family of copulas, called Archimedean, that satisfies very interesting properties (see among other [6,16]).

Although stochastic dominance and statistical preference have different interpretations, the former focusing on the marginal distributions and latter on the joint distribution, they have recently been investigated (see [8,23,22]). Those papers proved that, when comparing independent random variables, if they are ordered by first degree stochastic dominance, they are also ordered with respect to statistical preference. However, since statistical preference is a method that takes into account the dependence between the variables, the relationship between them should be investigated for non-independent random variables. Thus, our aim in this paper is to investigate under which conditions, based on the copula that links the variables, first degree stochastic dominance implies statistical preference.

The rest of the paper is organized as follows. After this introduction we start with some preliminaries: on the one hand we show the definitions and main properties of the stochastic orders we will use, stochastic dominance and statistical preference; on the other hand, we make an overview on the theory of copulas. Then, in Section 3 we look for a characterization of statistical preference for different kinds of random variables. We shall consider two cases: comonotone and countermonotone random variables that are either absolutely continuous or discrete with finite supports and absolutely continuous random variables coupled by an Archimedean copula. In Section 4 we will prove that, although first degree stochastic dominance does not imply statistical preference in general, it does for some kinds of random variables, while Section 5 shows that there is not a general relationship between the nth degree stochastic dominance and statistical preference. Finally, we show in Section 6 a decision making problem where our results can be applied.

In order to make the paper readable, we have moved the proofs of our original results in Sections 3 and 4, as well as technical lemmas, to Appendix.

2. Preliminaries

In this section we present the well-known concepts and results that will be useful in the rest of the paper. The first part introduces the stochastic orders we shall work with, stochastic dominance and statistical preference, while in the second part we make an overview on the theory of copulas.

2.1. Stochastic orders

Stochastic orders are methods that allow the comparison of random quantities. There is a huge amount of literature regarding stochastic orders, and many different methods have been proposed (see for instance [24,26]). In this work we shall focus on stochastic dominance and statistical preference.

Stochastic dominance is one of the most usual methods used in the literature for the pairwise comparison of random variables. The most common way of stochastic dominance is the first degree, that is based on the direct comparison of the cumulative distribution functions.

Definition 1. Let X and Y be two random variables, and let F_X and F_Y denote their respective cumulative distribution functions. X stochastically dominates Y by the first degree if:

$$F_X(t) \leq F_Y(t), \quad \text{for every } t \in \mathbb{R}. \tag{1}$$

It is usually denoted by $X \succeq_{FSD} Y$.

This stochastic order requires that one of the cumulative distribution functions must dominate the other one. However, Eq. (1) imposes a very strong condition. In fact, it is not usual that given two random variables, their respective cumulative distribution functions fulfill this condition. For this reason, such condition can be relaxed.

Definition 2. Let X and Y be two random variables, and let F_X and F_Y denote their respective cumulative distribution functions. X stochastically dominates Y by the second degree if:

$$G_{2,X}(t) = \int_{-\infty}^{t} F_X(t)\mathrm{d}t \leq \int_{-\infty}^{t} F_Y(t)\mathrm{d}t = G_{2,Y}(t), \tag{2}$$

for every $t \in \mathbb{R}$, whenever both integrals are well-defined. It is usually denoted by $X \succeq_{SSD} Y$.

Obviously, Eq. (2) is less restrictive than Eq. (1). In fact, as we will see in Remark 46 further on, it is possible to find random variables such that there is not first degree stochastic dominance, but there is second degree. However, it is also possible to find some pairs of random variables which are not ordered using the second degree stochastic dominance. For this reason, Eq. (2) can be recursively relaxed.