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Shrinkage estimation in spatial autoregressive model

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ABSTRACT

The paper considers spatial econometric model and presents a family of shrinkage estimators for the regression coefficients vector. The asymptotic distribution of the proposed family of estimators has been derived under the assumption that sample size is large. The risk properties of least squares estimator and proposed improved family of estimators have been investigated under quadratic loss function and dominance conditions have been obtained. For investigating the finite sample behavior of various estimators belonging to proposed family of shrinkage estimators, a simulation study has been carried out and results have been presented.

1. Introduction

In the recent years, there has been a growing interest in specification and developing inference procedures for spatial econometric models. For modeling causal relationships for spatially referenced data keeping in view the presence of spatial dependence in observations, these models either incorporate errors having spatial autocorrelation (spatial error model) or by including dependent variable having spatial autocorrelation (spatial lag model). The spatial weight matrix with known weights represents a priori understanding of the nature of spatial interdependence between different geographical regions or between different economic agents. For theoretical overviews of spatial econometrics one may refer to Anselin [1]. LeSage and Pace [8] provide introduction to spatial econometric modeling along with its various applications and discuss classical and Bayesian inference procedures for spatial autoregressive (SAR) model, spatial Durbin model (SDM), and spatial error model (SEM).

Stemming from the philosophy of Stein rule estimators (see [13,6]), the past few decades have seen emergence of a large body of literature that concentrates on estimators falling outside the traditional class of unbiased estimators. Stein-rule estimators have been extensively used for estimating the coefficients vector of a linear regression model with spherical disturbances and provide improvement over the OLS estimator in terms of quadratic loss function when number of explanatory variables in the model is at least three. The literature on Stein-rule estimators is documented by Judge et al. [7] and Hoffman [5]. Ullah and Ullah [14] proposed a family of double *k*-class estimators and studied its finite sample properties. Often, the assumption of spherical disturbances is a matter of simplicity rather than a representation of reality. Chaturvedi and Shukla [4] proposed a family of Stein-rule estimators for the coefficients vector of a linear regression model with non-spherical disturbances and derived its asymptotic distribution. They also demonstrated the dominance of the estimator over the feasible generalized least squares (FGLS) estimator under a quadratic loss function. Ohtani [9] considered

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an operational minimum mean squared error (MMSE) estimator for the coefficients vector of a linear model and investigated its finite sample properties. Ohtani [10] modified the minimum mean squared error estimator and proposed an operational MMSE estimator adjusted for degrees of freedom, termed as adjusted minimum mean squared error (AMMSE) estimator. They investigated the finite sample behavior of the estimator under the assumption of normality of disturbances. Shalabh et al. [12] investigated the properties of Stein rule estimator under balanced loss function and non-normality of disturbances. Considering the model with non-spherical disturbances, Wan and Chaturvedi [15,16] extended the results of Chaturvedi and Shukla for operational variants of minimum mean squared error estimator, adjusted minimum mean squared error estimator and family of double k-class estimators. Chaturvedi and Shalabh [3] investigated the properties of general double k-class estimators under balanced loss function when disturbances are non-spherical.

The present paper considers spatial autoregressive (SAR) model involving one period lag spatial dependent variable and proposes a general family of shrinkage estimators for the regression coefficients vector. The asymptotic distribution of the proposed family of estimators has been derived under the assumption that sample size is large. The risk properties of the least squares estimator and proposed improved family of estimators have been investigated under quadratic loss function and dominance conditions have been obtained. For investigating the finite sample behavior of various estimators belonging to the proposed family of shrinkage estimators, a simulation study has been carried out and results have been presented.

2. The spatial autoregressive model and estimators

Let us consider the following spatial autoregressive (SAR) model involving k explanatory variables:

$$y = \rho W y + X\beta + u \tag{2.1}$$

where y is a $n \times 1$ vector of observations on cross sectional dependent variables, X is a $n \times k$ matrix of observations on k explanatory variables, β is a $k \times 1$ vector of unknown regression coefficients, W is a $n \times n$ known matrix of spatial weights, ρ is autoregressive parameter, and u is a $n \times 1$ vector of disturbances assumed to follow a normal distribution with $E(u) = 0$ and $E(uu') = \sigma_u^2 I_n$.

When ρ is known, the OLS estimator of β is

$$\tilde{\beta} = \left(X'X\right)^{-1} X' \left(y - \rho W y\right). \tag{2.2}$$

When ρ is unknown, we replace it by its estimator, say $\hat{\rho}$, in (2.2) to obtain a feasible least squares estimator of β as

$$\hat{\beta} = \left(X'X\right)^{-1} X' \left(y - \hat{\rho} W y\right). \tag{2.3}$$

3. The class of shrinkage estimators and its asymptotic distribution

First we assume that ρ is known and define the following family of shrinkage estimators:

$$\begin{aligned} \tilde{\beta}_S &= \left[1 - \frac{r\left(\frac{\tilde{\beta}'X'X\tilde{\beta}}{v}\right)}{n\frac{\tilde{\beta}'X'X\tilde{\beta}}{v}}\right]\tilde{\beta} \\ &= \left[1 - \frac{1}{n}\frac{r\left(\tilde{\eta}\right)}{\tilde{\eta}}\right]\tilde{\beta} \end{aligned} \tag{3.1}$$

where

$$\begin{aligned} \tilde{\eta} &= \frac{\tilde{\beta}'X'X\tilde{\beta}}{v} \\ v &= \left(y - \rho W y - X\tilde{\beta}\right)'\left(y - \rho W y - X\tilde{\beta}\right). \end{aligned}$$

Further, the function $r(\tilde{\eta})$ is a continuous, bounded and differentiable function of $\tilde{\eta}$.

In particular if we select $r\left(\tilde{\eta}\right) = k_1$, where $k_1 \geq 0$ is a constant, it leads to the Stein rule estimator

$$\tilde{\beta}_{SR} = \left[1 - \frac{k_1}{n}\frac{v}{\tilde{\beta}'X'X\tilde{\beta}}\right]\tilde{\beta}. \tag{3.2}$$

The double k-class estimator (see [14]) for the parameter vector of SAR model can be obtained by setting

$$r\left(\tilde{\eta}\right) = \frac{nk_1\tilde{\beta}'X'X\tilde{\beta}}{\tilde{\beta}'X'X\tilde{\beta} + k_2 v}$$