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Bias-corrected estimation of stable tail dependence function

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ABSTRACT

We consider the estimation of the stable tail dependence function. We propose a bias-corrected estimator and we establish its asymptotic behaviour under suitable assumptions. The finite sample performance of the proposed estimator is evaluated by means of an extensive simulation study where a comparison with alternatives from the recent literature is provided.

1. Introduction and notations

Many problems involving extreme events are inherently multivariate. For instance, de Haan and de Ronde [5] estimate the probability that a storm will cause a sea wall near the town of Petten (the Netherlands) to collapse because of a dangerous combination of sea level and wave height. Other examples can be found in actuarial science, finance, environmental science and geology, to name but a few. A fundamental question that arises when studying more than one variable is that of extremal dependence. Similarly to classical statistics one can summarise extremal dependency in a number of well-chosen coefficients that give a representative picture of the dependency structure. Here, the prime example of such a dependency measure is the coefficient of tail dependence [15]. Alternatively, a full characterisation of the extremal dependence between variables can be obtained from functions like e.g. the stable tail dependence function, the spectral distribution function or the Pickands dependence function. We refer to Beirlant et al. [3] and de Haan and Ferreira [6], and the references therein, for more details. In this paper we will focus on bias-corrected estimation of the stable tail dependence function.

For any arbitrary dimension d, let $(X^{(1)}, \dots, X^{(d)})$ be a multivariate vector with continuous marginal cumulative distribution functions (cdfs) $F_1, \dots, F_d$. The stable tail dependence function is defined for each $x_i \in \mathbb{R}_+,\ i = 1, \dots, d$, as

$$\lim_{t\to\infty} t\mathbb{P}\left(1 - F_1(X^{(1)}) \leqslant t^{-1}x_1 \text{ or } \dots \text{ or } 1 - F_d(X^{(d)}) \leqslant t^{-1}x_d\right) = L(x_1, \dots, x_d)$$

which can be rewritten as

$$\lim_{t\to\infty} t\left[1 - F\left(F_1^{-1}(1 - t^{-1}x_1), \dots, F_d^{-1}(1 - t^{-1}x_d)\right)\right] = L(x_1, \dots, x_d) \tag{1}$$

where F is the multivariate distribution function of the vector $(X^{(1)}, \dots, X^{(d)})$.

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Now, consider a sample of size n drawn from F and an intermediate sequence $k = k_n$, i.e. $k \to \infty$ as $n \to \infty$ with $k/n \to 0$. Let us denote $\mathbf{x} = (x_1, \ldots, x_d)$ a vector of the positive quadrant $\mathbb{R}_+^d$ and $X_{k,n}^{(j)}$ the kth order statistic among n realisations of the margins $X^{(j)}$, $j = 1, \ldots, d$. The empirical estimator of L is then given by

$$\widehat{L}_k(\mathbf{x}) = \frac{1}{k} \sum_{i=1}^{n} \mathbb{1}_{\{X_i^{(1)} \geqslant X_{n-[kx_1]+1,n}^{(1)} \text{ or } \ldots \text{ or } X_i^{(d)} \geqslant X_{n-[kx_d]+1,n}^{(d)}\}}.$$

The asymptotic behaviour of this estimator was first studied by Huang [14]; see also [8,6]. As is common in extreme value statistics, the empirical estimator $\widehat{L}_k(\mathbf{x})$ is affected by bias, which often complicates its application in practice. This bias-issue will be addressed in the present paper.

In the univariate framework there are numerous contributions to the bias-corrected estimation of the extreme value index and tail probabilities. Typically, the bias reduction of estimators for tail parameters is obtained by taking the second order structure of an extreme value model explicitly into account in the estimation stage. We refer here to Beirlant et al. [1], Feuerverger and Hall [10], Matthys and Beirlant [16], and more recently, Gomes et al. [13] and Caeiro et al. [4]. In the bivariate framework some attention has been paid to bias-corrected estimation of the coefficient of tail dependence η. Goegebeur and Guillou [12] obtained the bias correction by a properly weighted sum of two biased estimators, whereas Beirlant et al. [2] fitted the extended Pareto distribution to properly transformed bivariate observations. Recently, a robust and bias-corrected estimator for η was introduced by Dutang et al. [9]. For what concerns the stable tail dependence function we are only aware of the estimator recently proposed by Fougères et al. [11].

For the sequel, in order to study the behaviour of $\widehat{L}_k(\mathbf{x})$ or a function of it, we need to assume some conditions mentioned below and well-known in the extreme value framework:

First order condition: The limit in (1) exists and the convergence is uniform on $[0, T]^d$ for $T > 0$;

Second order condition: There exist a positive function α such that $\alpha(t) \to 0$ as $t \to \infty$ and a non null function M such that for all $\mathbf{x}$ with positive coordinates

$$\lim_{t\to\infty} \frac{1}{\alpha(t)} \left\{ t \left[1 - F\left(F_1^{-1}(1 - t^{-1}x_1), \ldots, F_d^{-1}(1 - t^{-1}x_d) \right) \right] - L(\mathbf{x}) \right\} = M(\mathbf{x}), \tag{2}$$

uniformly on $[0, T]^d$ for $T > 0$;

Third order condition: There exist a positive function β such that $\beta(t) \to 0$ as $t \to \infty$ and a non null function N such that for all $\mathbf{x}$ with positive coordinates

$$\lim_{t\to\infty} \frac{1}{\beta(t)} \left\{ \frac{t \left[1 - F\left(F_1^{-1}(1 - t^{-1}x_1), \ldots, F_d^{-1}(1 - t^{-1}x_d) \right) \right] - L(\mathbf{x})}{\alpha(t)} - M(\mathbf{x}) \right\} = N(\mathbf{x}), \tag{3}$$

uniformly on $[0, T]^d$ for $T > 0$. This requires that N is not a multiple of M.

Note that these assumptions imply that the functions α and β are both regularly varying with indices ρ and ρ' respectively which are non positive. In the sequel we assume that both indices are negative. Remark also that the functions L, M and N have an homogeneity property, that is $L(a\mathbf{x}) = aL(\mathbf{x})$, $M(a\mathbf{x}) = a^{1-\rho}M(\mathbf{x})$ and $N(a\mathbf{x}) = a^{1-\rho-\rho'}N(\mathbf{x})$ for a positive scale parameter a.

In this paper, based on the process representation for the empirical estimator $\widehat{L}_k$ given in Proposition 2 in [11], we introduce a novel bias correction procedure. Indeed, we propose to estimate the bias directly and then to subtract it from our uncorrected kernel estimator. This is an alternative to the approach proposed by Fougères et al. [11] where differences between estimators of L are used to eliminate the bias. Moreover, in the spirit of Gomes et al. [13], we show that using the present approach the bias is decreased while keeping the asymptotic variance at the level of the uncorrected kernel estimator. These theoretical results are also complemented by improved finite sample behaviour.

The remainder of our paper is organised as follows. In the next section we introduce our estimators for $L(\mathbf{x})$, as well as for the second order quantities ρ and α, and study their asymptotic properties. The finite sample performance of the proposed bias-corrected estimator and of some estimators from the recent literature are evaluated by a simulation experiment in Section 3. The proofs of all results are given in the Appendix.

2. Estimators and asymptotic properties

Consider now the rescaled version

$$\widehat{L}_{k,a}(\mathbf{x}) := a^{-1}\widehat{L}_k(a\mathbf{x})$$

for a positive scale parameter a. Our first aim is to look at the behaviour of

$$\widetilde{L}_k(\mathbf{x}) := \frac{1}{k} \sum_{j=1}^{k} K(a_j)\widehat{L}_{k,a_j}(\mathbf{x})$$

where $a_j := \frac{j}{k+1}$, $j = 1, \ldots, k$, and K is a function defined on $(0, 1)$ which is positive and such that $\int_0^1 K(u)du = 1$. This function is called a kernel function in the sequel. Let $\mathbf{e}_j$ be a d-vector with zeros, except for position j where it is one.