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Strong consistency of the distribution estimator in the nonlinear autoregressive time series

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ABSTRACT

This paper considers the uniform strong consistency of the error cumulative distribution function (CDF) estimator. Under appropriate assumptions, the classical Glivenko–Cantelli Theorem is obtained for the residual based empirical error CDF in the nonlinear autoregressive time series.

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1. Introduction

Let $\{X_i, i = 0, \pm 1, \pm 2, \ldots\}$ be a strictly stationary process of real random variables obeying the ergodic model

$$X_i = r_\theta(X_{i-1}, \ldots, X_{i-p}) + \varepsilon_i \tag{1.1}$$

for some $\theta = (\theta_1, \ldots, \theta_q)^\top \in \Theta \subset R^q$, where $r_\theta, \theta \in \Theta$, is a family of known measurable functions from $R^p \to R$. Moreover, the errors $\{\varepsilon_i\}$ are assumed to be i.i.d. random variables with mean being 0 and common cumulative distribution function (CDF) F, and $X_{i-1}, \ldots, X_{i-p}$ are independent of $\{\varepsilon_i, \ i = 0, \pm 1, \pm 2, \ldots\}$.

The monograph by Tong [8] represents a good account of nonlinear time series models of type (1.1). The error density estimation in model (1.1) has been considered in Liebscher [7], Cheng and Sun [4], and Cheng [3]. The strong consistency and asymptotic normality of the error density estimator are obtained in Liebscher [7]. Cheng and Sun [4] consider the problem of fitting an error density to the goodness-of-fit test of the errors, and obtain the asymptotic properties of the test. Cheng [3] develops the asymptotic distribution of the maximum of a suitably normalized deviation of the density estimator from the expectation of the kernel error density (based on the true error), which is shown to be the same as in the case of the one sample set up, which is given in Bickel and Rosenblatt [2]. Here, we will continue to develop the uniform strong consistency of the CDF estimator in model (1.1). For the Lipschitz continuous CDF F, we shall extend the classical Glivenko–Cantelli Theorem to the residual based empirical error CDF for the nonlinear autoregressive model (1.1).

The paper is organized as follows. In Section 2 we first introduce some basic assumptions on the model (1.1), and the estimator $\hat{\theta}$ for θ; define the CDF estimator $\hat{F}$ based on the residuals of model (1.1). Then we describe the main result. Section 3 provides the details of the proof.

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2. Basic assumptions and estimators

In this section we first introduce some basic assumptions.

For the autoregression function r_θ in model (1.1), we have the following assumptions.

Assumption 1. Let $U \subset \Theta \subset R^q$ be an open neighborhood of θ. We assume there exists a function M such that, for all $y \in R^p,\ \theta^* = (\theta_1^*, \ldots, \theta_q^*) \in U,\ j = 1, \ldots, q,$

$$\left| \frac{\partial}{\partial \theta_j} r_{\theta^*}(y) \right| \le M(y),$$

and $EM(X_{i-1}, \ldots, X_{i-p}) < +\infty$.

For $1 \le i \le n$, set

$$M_i = M(X_{i-1}, \ldots, X_{i-p}).$$

Remark 1. It is easy to see that ε_i and M_i are independent, and

$$E(M_i) = E\Big(M(X_1, \ldots, X_p)\Big) < \infty.$$

By Theorem 2.3 in Fan and Yao [5], we also have that

$$\frac{1}{n}\sum_{i=1}^{n} M_i \to E(M_1), \quad \text{a.s.} \tag{2.1}$$

Assumption 2. Suppose that we observe $X_{1-p}, \ldots, X_n$. Let $\hat{\theta} = (\hat{\theta}_1, \ldots, \hat{\theta}_q)$ be an estimator for θ with the property: $\hat{\theta}$ being a strong consistent estimator (for θ) which satisfies the law of iterated logarithm.

Therefore, under Assumption 2, there exists a constant $C_1 (0 < C_1 < \infty)$ such that

$$\limsup_{n\to\infty} \sqrt{\frac{n}{\log\log n}} \left|\hat{\theta} - \theta\right| \le C_1 \quad \text{a.s.}, \tag{2.2}$$

where $\left|\hat{\theta} - \theta\right| = \sqrt{\sum_{j=1}^{q} (\hat{\theta}_j - \theta_j)^2}$.

Remark 2. The above assumption on $\hat{\theta}$ holds for least square estimator under certain conditions (see Klimko and Nelson [6]). This assumption is also used in Liebscher [7].

Based on the estimator $\hat{\theta}$, we define the residuals

$$\hat{\varepsilon}_i := X_i - r_{\hat{\theta}}(X_{i-1}, \ldots, X_{i-p}), \quad i = 1, 2, \ldots, n. \tag{2.3}$$

Note that $\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n$ are i.i.d. with the unknown CDF F. Let F_n denote the empirical distribution function, i.e.,

$$F_n(t) = \frac{1}{n}\sum_{i=1}^{n} I(\varepsilon_i \le t), \quad t \in R^1,$$

where I denotes the indicator function.

The classical Glivenko–Cantelli theorem says that $F_n(t)$ converges almost surely (a.s.) to $F(t)$ uniformly in $t \in R^1$, i.e.,

$$\sup_{t\in R^1} |F_n(t) - F(t)| \to 0, \quad \text{a.s.} \tag{2.4}$$

Notice that F_n is infeasible for model (1.1), since $\varepsilon_i,\ 1 \le i \le n$ are not observable. Here, based on residuals $\hat{\varepsilon}_i$ $(1 \le i \le n)$, we construct empirical distribution function $\hat{F}_n$ as follows:

$$\hat{F}_n(t) = \frac{1}{n}\sum_{i=1}^{n} I(\hat{\varepsilon}_i \le t), \quad t \in R^1.$$

We will consider the uniform strong convergence of $\hat{F}_n$ for F. The main result is the following Glivenko–Cantelli Theorem for the estimator $\hat{F}_n$.

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