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Matrix variate slash distribution

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ABSTRACT

In this paper, we introduce a matrix variate slash distribution as a scale mixture of the matrix variate normal and the uniform distributions. We study some properties of the proposed distribution and give maximum likelihood (ML) estimators of its parameters using EM algorithm. We provide an iteratively reweighting algorithm to compute the ML estimates. Also, we give a small simulation study to show performance of the algorithm.

1. Introduction

Let $Z \sim N(0, 1)$ and $U \sim U(0, 1)$ and assume that Z and U are independent. Then the random variable $ZU^{-\frac{1}{q}}$, $q > 0$ will have slash distribution [12,13,15,18]. When $q = 1$, we get standard slash distribution and when $q \to \infty$, we get normal distribution. Some extension of the slash distribution can be found in [5–9,16]. The probability density function (pdf) of the univariate slash distribution is given by

$$h(x; q) = q \int_0^1 t^q \phi(xt)\, dt \tag{1}$$

where q is a shape parameter and $\phi(.)$ denotes the standard normal density function. The univariate slash distribution has important role in robust statistical analysis as it has heavier tails than the normal distribution (for further details, see [14,18]).

Wang and Genton [19] introduced multivariate slash and multivariate skew–slash distributions. Gomez et al. [7] introduced extension of univariate and multivariate slash distributions as a scale mixture of elliptically contour distributions. Arslan and Genc [3] introduced a generalization of multivariate slash distribution by using Kotz-type distribution. Also, Arslan [1,2] defined a new extension of skew–slash distribution in multivariate setting and provided ML estimators for the parameters of the proposed distribution based on the EM algorithm. Recently, Reyes et al. [17] also introduced the modified slash distribution in univariate and multivariate settings as a scale mixture of normal distribution and exponential

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distribution with the scale parameter 2. They gave some distributional properties and parameter estimation. They showed that the modified slash distribution has heavier tails than the ordinary slash distribution. One can see Reyes et al. [17] for further details and the application on the modified slash distribution.

Wang and Genton [19] defined multivariate slash distribution as follows.

Definition 1. Let $\mathbf{Z} \sim N_n(0, I_n)$ be an n-dimensional random variable, U be a scale random variable which has uniform distribution on interval (0, 1). Also, $\mathbf{Z}$ and U are independent random variables. Then, the random variable $\mathbf{X} = \mu + \Sigma^{1/2}\mathbf{Z}U^{-1/q}$ has a multivariate slash distribution and the density function of $\mathbf{X}$ is given as follows

$$p(x; q, \mu, \Sigma) = \begin{cases} \dfrac{q2^{(q+n)/2-1}\,|\Sigma|^{-1/2}\,\gamma((q+n)/2, \left[(x-\mu)'\Sigma^{-1}(x-\mu)\right]/2)}{(2\pi)^{n/2}\left[(x-\mu)'\Sigma^{-1}(x-\mu)\right]^{(q+n)/2}}, & x \neq \mu \\ \dfrac{q}{(2\pi)^{n/2}(q+n)}\,|\Sigma|^{-1/2}, \quad x = \mu \end{cases} \tag{2}$$

where γ is an incomplete gamma function, $q > 0$ is the shape parameter, $\mu \in R^n$ is the location parameter and Σ is the positive definite scatter matrix.

In this paper, we introduce a matrix variate version of the slash distribution given in the above definition. The new distribution is defined as a scale mixture of the matrix variate normal distribution and the uniform distribution. Note that the details about the matrix variate normal distribution and the scale mixture of matrix normal distribution can be found in [10,11]. We give the density function of the proposed distribution and show that the linear transformation of a matrix variate slash distributed random matrix has again a matrix variate slash distribution.

The paper is organized as follows. In Section 2, we give the definition of the matrix variate slash distribution and study some of its properties. In Section 3, we provide a procedure to obtain the ML estimators for the parameters of the proposed distribution. In Section 4, we give a small simulation study to show the performance of the proposed algorithm to compute the estimates.

2. Matrix variate slash distribution

In this section, we define matrix variate slash distribution and derive its pdf. Also, we give some distributional properties of the proposed distribution and show that matrix variate slash distribution is invariant under linear transformation.

Definition 2. The random variable $\mathbf{X} \in R^{n\times p}, n \geq 1, p \geq 1$ is said to have a scale mixture of matrix variate normal distributions if

$$\mathbf{X} = M + \Sigma^{\frac{1}{2}}\mathbf{Z}U^{-\frac{1}{q}}\Psi^{\frac{1}{2}} \tag{3}$$

where $\mathbf{Z} \sim N_{n,p}\left(0, I_n, I_p\right)$ and $U > 0$ scalar valued random variable, independent of $\mathbf{Z}$. Here $M \in R^{n\times p}$ is a location matrix and $\Sigma^{\frac{1}{2}}$ and $\Psi^{\frac{1}{2}}$ are square roots of positive definite scatter matrices Σ and Ψ, respectively.

One can easily see that conditional distribution of $\mathbf{X}$ given $U = u$ is $N_{n,p}\left(M, u^{-\frac{2}{q}}\Sigma, \Psi\right)$. It can be easily obtained that if $\mathbf{Z} \sim N_{n,p}\left(0, I_n, I_p\right)$ the expectation and the covariance of $\mathbf{Z}$ are

$$E(\mathbf{Z}) = 0$$
$$Cov(\mathbf{Z}) = I_n \otimes I_p.$$

Using these, the expectation and the covariance of $\mathbf{X}$ given in (3) can be obtained as

$$E(\mathbf{X}) = M + \Sigma^{\frac{1}{2}}E(\mathbf{Z})E\left(U^{-\frac{1}{q}}\right)\Psi^{\frac{1}{2}} = M \tag{4}$$

$$\begin{aligned} Cov(\mathbf{X}) &= E_U(Cov(\mathbf{X}|U)) \\ &= E_U\left(u^{-\frac{2}{q}}\Sigma \otimes \Psi\right) \\ &= E_U\left(u^{-\frac{2}{q}}\right)(\Sigma \otimes \Psi). \end{aligned} \tag{5}$$

Proposition 1. *Let $\mathbf{Z} \sim N_{n,p}\left(0, I_n, I_p\right)$ and $U \sim U(0, 1)$, and assume that $\mathbf{Z}$ and U are independent. Then, the distribution of the random matrix $\mathbf{X}$ given in (3) has the following density function*

$$f_{\mathbf{X}}(X; M, \Sigma, \Psi, q) = \begin{cases} \dfrac{q2^{\frac{np+q}{2}-1}\,|\Sigma|^{-\frac{p}{2}}\,|\Psi|^{-\frac{n}{2}}\,\gamma\left(\frac{np+q}{2}, \frac{1}{2}tr\left\{\Sigma^{-1}(X-M)\Psi^{-1}(X-M)'\right\}\right)}{(2\pi)^{\frac{1}{2}np}\left[tr\left\{\Sigma^{-1}(X-M)\Psi^{-1}(X-M)'\right\}\right]^{\frac{np+q}{2}}}, & X \neq M \\ \dfrac{|\Sigma|^{-\frac{p}{2}}\,|\Psi|^{-\frac{n}{2}}}{(2\pi)^{\frac{1}{2}np}}\dfrac{q}{np+q}, \quad X = M \end{cases} \tag{6}$$