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Third-order local power properties of tests for a composite hypothesis, II

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ABSTRACT

The Bartlett-type adjustment is a higher-order asymptotic method for improving the chi-squared approximation to the null distributions of various test statistics, which ensures that the resulting test has size $\alpha + o(N^{-1})$, where $0 < \alpha < 1$ is the significance level and N is the sample size. We continue our recent works on the third-order average local power properties of several Bartlett-type adjusted tests. Strengthening the results in the 1990s, the third-order optimality of the adjusted Rao test in a sense has been established even if both the interest parameter and the nuisance parameter are multi-dimensional. We briefly discuss adjusted profile likelihood inference for handling the nuisance parameter.

1. Introduction

We continue our recent works [10–13] on higher-order asymptotic theory of several statistics for testing a composite hypothesis about a subvector of parameters. Here, the $N^{-i/2}$-term is referred to as being the $(i+1)$th-order, where N is the sample size. A detailed historical review for comparing higher-order local powers, starting from the second-order local power analyses [22], is omitted here to save space; see Kakizawa [13] and the references cited therein.

In the absence of nuisance parameter, Mukerjee [17,20] established that Rao's (score) test under the third-order conditions of size and local unbiasedness has the third-order optimality in terms of average local power criterion. Mukerjee [21] additionally showed that Rao's test even in the original form (not being adjusted for local unbiasedness and only the size condition is being retained) has the third-order optimality, where we observe that 'the test in the original form' is nothing but the size-adjusted test with substitution of Cornish–Fisher's type expansion for the percentile. On the other hand, not much work has yet been reported on the third-order local power properties in the presence of nuisance parameter, except that Mukerjee [16,18] attempted to discuss the third-order optimality of Rao's (adjusted) test under the assumption of the global parameter orthogonality for the situation where both the interest parameter and the nuisance parameter are scalar. He mentioned that the same argument is applicable even when the nuisance parameter is multi-dimensional.

The present paper addresses the comments in the review paper [20] that

> *if both the interest parameter and the nuisance parameter be multi-dimensional, then, as noted in Cox and Reid, one may not in general be able to achieve an orthogonal parameterization. Anyway, it is strongly believed that the results discussed here should have their counterparts even in such a situation.*

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As a companion paper to Kakizawa [13], we are primarily concerned with the third-order local power properties of several Bartlett-type adjusted tests. The Bartlett-type adjustment dates back to different three methods proposed by Chandra and Mukerjee [2], Cordeiro and Ferrari [3], and Taniguchi [25] in alphabetical order. It is a higher-order asymptotic method for improving the chi-squared approximation to the null distributions of various test statistics, which ensures that the resulting test has size $\alpha + o(N^{-1})$, as in the size-adjusted test based on Cornish–Fisher's type expansion for the percentile, where $0 < \alpha < 1$ is the significance level. Rao and Mukerjee [23,24] have compared the third-order point-by-point local powers of three Bartlett-type adjustments [2,3,25] for the simple hypothesis on a scalar parameter. In recent years, there have been renewed interests [9–11] due to the existence of infinitely many Bartlett-type adjustments for the multi-parameter hypothesis testing.

By constructions (see Definitions 1 and 2 in Section 2), it will be convenient for us to define two types separately. One is the generalized Bartlett-type adjustment (for short GB). The other is the generalized Cordeiro–Ferrari Bartlett-type adjustment (for short GCF). We denote by $T^{\mathrm{GB}(N)}$ and $T^{\mathrm{GCF}(N)}$ the GB and GCF adjustments for a likelihood-based test statistic $T^{(N)} \in \mathcal{T}_{N,3}$ under consideration (see (3)). Kakizawa [13] derived the third-order average local power of the GB-adjusted test $T^{\mathrm{GB}(N)} > \chi^2_{p_1,\alpha}$, where $\chi^2_{p_1,\alpha}$ is the upper α-point of the central chi-squared distribution with p_1 degrees of freedom, and then established that even if both the interest parameter and the nuisance parameter are multi-dimensional, the GB-adjusted Rao test has the third-order optimality. So, Mukerjee's conjectural statement, as mentioned before, may be solved in a sense. However, we know that Rao's test statistic has many variants; e.g. $\mathrm{R}^{(N)}$ and $\mathrm{MR}^{(N)}$ (see (2)), for which the adjusted tests $\mathrm{R}^{\mathrm{GB}(N)} > \chi^2_{p_1,\alpha}$ and $\mathrm{MR}^{\mathrm{GB}(N)} > \chi^2_{p_1,\alpha}$ have the identical average local power up to the third-order. That is, the GB adjustment smooths out the distinctive features between $\mathrm{R}^{(N)}$ and $\mathrm{MR}^{(N)}$, and hence it may be more interesting to compare them (announced at the end of Section 4 of [13]). This is the reason why we need to have further discussion, on the basis of the GCF adjustment.

The contribution of the present paper is three fold. First, our results allow both the interest parameter and the nuisance parameter to be multi-dimensional, for which there is no assumption regarding the global parameter orthogonality. Second, we elucidate that the adjusted Rao tests $\mathrm{R}^{\mathrm{GCF}(N)} > \chi^2_{p_1,\alpha}$ and $\mathrm{MR}^{\mathrm{GCF}(N)} > \chi^2_{p_1,\alpha}$ are, generally, discriminated in terms of the third-order average local power, and that the former test $\mathrm{R}^{\mathrm{GCF}(N)} > \chi^2_{p_1,\alpha}$ has the third-order optimality in a large class of the GB and GCF-adjusted tests. Third, we briefly discuss adjusted profile likelihood inference (e.g. [4,6]), which represents an important tool for handling the nuisance parameter.

Although we focus on the i.i.d. case for notational simplicity, we arrive at the same conclusions even in a non-i.i.d. case where some regularity conditions are met for the log-likelihood derivatives according to the situations under consideration. We retain throughout this paper the notation and conventions of Kakizawa [13] (see also [10,11]). The rest of this paper is organized as follows. Section 2 contains the notation to be used throughout this paper. Section 3 derives an asymptotic expansion formula for the (average) local power of the GCF-adjusted test $T^{\mathrm{GCF}(N)} > \chi^2_{p_1,\alpha}$. Section 4 describes main results. Concluding remarks are given in Section 5.

2. Bartlett-type adjustments

2.1. Notation

We denote by $P^{(N)}_{\theta}$ the $\boldsymbol{\theta}$-distribution of $\mathbf{X}_1, \ldots, \mathbf{X}_N$, which are i.i.d. random vectors (taking values of $\mathbf{R}^{d_X}$) according to a density $f(\mathbf{x}, \boldsymbol{\theta})$, $\boldsymbol{\theta} \in \boldsymbol{\Theta} \subset \mathbf{R}^p$. For any sequence $\{Y^{(N)}\}_{N\geq 1}$ of random variables having the form $Y^{(N)} = g_N(\mathbf{X}_1, \ldots, \mathbf{X}_N)$, we use the pointwise notation $Y^{(N)} = \mathrm{o}^{(N)}_{\theta}(q, \beta)$ under $P^{(N)}_{\theta}$, if $P^{(N)}_{\theta}[|Y^{(N)}| > d(\log N)^{\beta}] = \mathrm{o}(N^{-q})$ as $N \to \infty$ for some $d > 0$, $q \geq 0$, and $\beta \geq 0$. In what follows, we assume the same regularity conditions as in Kakizawa [13]. Suppose that the parameter $\boldsymbol{\theta} = (\theta_1, \ldots, \theta_p)'$ is composed of two parts, a parameter of interest $\boldsymbol{\theta}_{(1)} = (\theta_1, \ldots, \theta_{p_1})'$ and a nuisance parameter $\boldsymbol{\theta}_{(2)} = (\theta_{p_1+1}, \ldots, \theta_{p_1+p_2})'$; $\boldsymbol{\theta} = (\boldsymbol{\theta}'_{(1)}, \boldsymbol{\theta}'_{(2)})' \in \boldsymbol{\Theta} = \boldsymbol{\Theta}_{(1)} \times \boldsymbol{\Theta}_{(2)}$ (say), where $p = p_1 + p_2$. We write $\mathcal{L}^{(N)}(\boldsymbol{\theta}) = \sum_{i=1}^N \log f(\mathbf{X}_i, \boldsymbol{\theta})$. We want to test a composite hypothesis $\boldsymbol{\theta}_{(1)} = \boldsymbol{\theta}_{(1)0}$ against $\boldsymbol{\theta}_{(1)} \neq \boldsymbol{\theta}_{(1)0}$, where $\boldsymbol{\theta}_{(1)0} \in \boldsymbol{\Theta}_{(1)}$ is specified while $\boldsymbol{\theta}_{(2)} \in \boldsymbol{\Theta}_{(2)}$ remains unspecified. Let $\widehat{\boldsymbol{\theta}}^{(N)}_{\mathrm{ML}} \in \boldsymbol{\Theta}$ be the (unrestricted) maximum likelihood estimator (MLE) of $\boldsymbol{\theta}$, and let $\widetilde{\boldsymbol{\theta}}^{(N)}_{(2)\mathrm{ML}} \in \boldsymbol{\Theta}_{(2)}$ be the restricted MLE of $\boldsymbol{\theta}_{(2)}$ under the constraint $\boldsymbol{\theta}_{(1)} = \boldsymbol{\theta}_{(1)0}$. We write

$$\widetilde{\boldsymbol{\theta}}^{(N)}_{\mathrm{ML}} = \begin{pmatrix} \boldsymbol{\theta}_{(1)0} \\ \widetilde{\boldsymbol{\theta}}^{(N)}_{(2)\mathrm{ML}} \end{pmatrix}, \qquad \widehat{\boldsymbol{\theta}}^{(N)}_{\mathrm{ML}} = \begin{pmatrix} \widehat{\boldsymbol{\theta}}^{(N)}_{(1)\mathrm{ML}} \\ \widehat{\boldsymbol{\theta}}^{(N)}_{(2)\mathrm{ML}} \end{pmatrix}, \quad \text{and} \quad \boldsymbol{\theta}^{\dagger} = \begin{pmatrix} \boldsymbol{\theta}_{(1)0} \\ \boldsymbol{\theta}^{\dagger}_{(2)} \end{pmatrix} \in \boldsymbol{\Theta},$$

with $\boldsymbol{\theta}^{\dagger}_{(2)}$ being the irrelevant true value of the nuisance parameter $\boldsymbol{\theta}_{(2)}$. For any (nonrandom/random) scalar or vector or matrix function $Q(\cdot)$, we use the notation $\widehat{Q}$, $\widetilde{Q}$, and Q instead of $Q(\widehat{\boldsymbol{\theta}}^{(N)}_{\mathrm{ML}})$, $Q(\widetilde{\boldsymbol{\theta}}^{(N)}_{\mathrm{ML}})$, and $Q(\boldsymbol{\theta}^{\dagger})$, respectively.

The Rth partial derivative of the log density $\log f(\mathbf{x}, \boldsymbol{\theta})$ with respect to $\boldsymbol{\theta}$ is denoted by

$$\ell_{j_1\cdots j_R}(\mathbf{x}, \boldsymbol{\theta}) = \frac{\partial}{\partial\theta_{j_1}} \cdots \frac{\partial}{\partial\theta_{j_R}} \log f(\mathbf{x}, \boldsymbol{\theta})$$