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Bootstrap for dependent Hilbert space-valued random variables with application to von Mises statistics[☆]

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ABSTRACT

Statistical methods for functional data are of interest for many applications. In this paper, we prove a central limit theorem for random variables taking their values in a Hilbert space. The random variables are assumed to be weakly dependent in the sense of near epoch dependence, where the underlying process fulfills some mixing conditions. As parametric inference in an infinite dimensional space is difficult, we show that the nonoverlapping block bootstrap is consistent. Furthermore, we show how these results can be used for degenerate von Mises-statistics.

1. Introduction and main results

1.1. Introduction

In many medical and biological problems, when you are dealing with genomics, transcriptomics and proteomics data, the number of variables may be much larger than the number of subjects and traditional parametric methods cannot be used while in contrast particular nonparametric methods can, see Marozzi [28]. Imaging methods in medicine like functional magnetic resonance imaging lead to function valued time series, see Lange [21], Aston and Kirch [2]. Furthermore, observations measured on a fine time grid can be often treated as a sequence of observed functions on longer periods instead of a seasonal time series with high resolution. Examples include environmental data, see Hörmann and Kokoszka [18], or medical data, see Cuevas, Febrero, and Fraiman [10].

The first aim of this paper is to establish a bootstrap method for dependent Hilbert space-valued random variables. Assume that a sequence of Hilbert space-valued random variables $(X_n)_{n\in\mathbb{Z}}$ with mean μ satisfies a central limit, i.e. for any Borel set A with $P(N \in \partial A) = 0$ we have the convergence

$$\left| P\left(\frac{1}{\sqrt{n}}\sum_{i=1}^{n}(X_i-\mu)\in A\right) - P(N\in A)\right| \to 0 \tag{1}$$

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as $n \to \infty$, where N is a centered Gaussian Hilbert space-valued random variable with mean zero and covariance operator V.

In functional data analysis in order to make some statistical inferences (construct confidence regions and tests) on an unknown parameter μ that is asymptotically normal, one needs to calculate probabilities $P(N \in A)$ for different sets A. Such probabilities are not easy to calculate even in the case when the covariance operator V is known and the set A has a simple structure. This probability depends on infinite number of eigenvalues and eigenfunctions of the operator V. The situation becomes more complicated when A is a "bad" Borel set and V is unknown and has to be estimated. Thus unlike the one dimensional case where, in general, one can use both the central limit and as an alternative the bootstrap, in Hilbert space the bootstrap becomes more important.

Consistency of the bootstrap for the sample mean of the independent random variables with values in Banach spaces were established by Giné and Zinn [14]. To the best of our knowledge there is only one paper by Politis and Romano [32] in which the validity of the stationary bootstrap for dependent Hilbert space valued random variables was proved. This is also stated in review papers by McMurry and Politis [29] and Gonçalves and Politis [16]. Up to now it is an open problem whether the bootstrap methods with fixed block length can be used in Hilbert space. We will establish a strong consistency of nonoverlapping block bootstrap for the sample mean of dependent Hilbert space-valued random variables. We assume that the time series is near epoch dependent on an unobserved underlying process which is absolutely regular. This is a more general model than the strong mixing assumed by Politis and Romano [32]. Also, their result is restricted to bounded random variables.

The second aim of the paper is a bootstrap for von Mises statistics of dependent observations. Bootstrap for von Mises and U-statistics with nondegenerate kernel were studied by Arcones and Giné [1], Dehling and Mikosch [12] in the case of independent observations and by Leucht and Neumann [24,25], Leucht [23] in the case of dependent observations. We want to show that the validity of the bootstrap for von Mises and U-statistics with degenerate kernel can be proved using bootstrap for Hilbert space-valued random variables.

The paper is organized as follows: In the next subsection we will formulate the central limit theorem for stationary sequences of near epoch dependent Hilbert space-valued random variables that will be used in the proof of the next theorem. The central limit theorem for mixing Hilbert space-valued random variables was studied in Kuelbs and Philipp [20], Dehling [11], Maltsev and Ostrovskii [27], Zhurbenko and Zuparov [38]. Under near epoch dependence, a central limit theorem was proved by Chen and White, a weak invariance principle was given by Berkes, Horváth, and Rice [3]. Section 1.3 is devoted to the bootstrap for Hilbert space-valued random variables. In this section we will formulate a theorem which establishes the strong consistency of the nonoverlapping block bootstrap for the sample mean of near epoch dependent Hilbert space-valued random variables. In Section 1.4 we will give a theorem on the validity of the bootstrap for von Mises statistics of near epoch dependent observations. And finally proofs will be given in Section 3 where we will use preliminary results from Section 2.

1.2. Central limit theorem for hilbert space-valued functionals of mixing random variables

Let H be a separable Hilbert space with the inner product $\langle \cdot, \cdot \rangle$ and norm $\|\cdot\| = \sqrt{\langle \cdot, \cdot \rangle}$. Consider a two-sided, stationary sequence $(\xi_n)_{n\in\mathbb{Z}}$ of random variables with values in a separable measurable space S. We say that $(X_n)_{n\in\mathbb{Z}}$ is a functional of $(\xi_n)_{n\in\mathbb{Z}}$ if there exists a measurable function $f : S^{\mathbb{Z}} \to H$ such that

$$X_n = f\left((\xi_{n+i})_{i\in\mathbb{N}}\right). \tag{2}$$

We say that f is a 1-approximating functional (or near epoch dependent) if there exists a sequence $(a_m)_{m\in\mathbb{N}}$ with $a_m \to 0$ as $m \to 0$ and for every m a function $f_m : S^{2m+1} \to H$ such that

$$E\, \|X_0 - f_m(\xi_{-m}, \dots, \xi_m)\| \le a_m \quad \text{for all } m \in \mathbb{N}. \tag{3}$$

As convergence in L_2 implies convergence in L_1, the 1-approximating property is more general than L_2 near epoch dependence, which is used more often in the literature. In what follows, we will assume that the sequence $(\xi_n)_{n\in\mathbb{Z}}$ is absolutely regular (β-mixing). We define the coefficients of absolute regularity $(\beta_m)_{m\in\mathbb{Z}}$ by

$$\beta_m = E\Big[\sup_{A\in\mathcal{F}_m^{\infty}} \big(P(A|\mathcal{F}_{-\infty}^{0}) - P(A)\big)\Big], \tag{4}$$

where $\mathcal{F}_a^b$ is the σ-field generated by $\xi_a, \dots, \xi_b$, and call the sequence $(\xi_n)_{n\in\mathbb{Z}}$ absolutely regular if $\beta_m \to 0$ as $m \to \infty$. For more details on absolute regularity, see the book of Bradley [6]. Approximating functionals of underlying absolutely regular sequences cover many examples of times series, e.g. linear processes or expanding dynamical systems, see Hofbauer and Keller [17].

The first result of this paper is a central limit theorem for approximating functionals of absolutely regular sequences: