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Comparisons of variance estimators in a misspecified linear model with elliptically contoured errors

Guikai Hu [a,b,*], Shenghua Yu [c], Han Luo [b]

[a] School of Science, East China Institute of Technology, Nanchang 330013, China
[b] School of Mathematics and Econometrics, Hunan University, Changsha 410082, China
[c] School of Economics and Trade, Hunan University, Changsha 410079, China

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In a misspecified linear regression model with elliptically contoured errors, the exact biases and risks of least squares, restricted least squares, preliminary test and Stein-type estimators of the error variance are derived. Also, we compare the risk performances of the underlying estimators and give the dominance pictures for them. It is shown that the risk of preliminary test estimator attains the smallest value if the critical value equals one. Moreover, we give a bootstrap procedure for estimating the risks of proposed estimators in order to overcome the difficulty of computing the exact risks when the sample size becomes larger.

1. Introduction

Consider the linear regression model:

$$y = X_1\beta_1 + X_2\beta_2 + \varepsilon, \tag{1.1}$$

where y is an $n \times 1$ observable random vector, $\beta_1 \in R^{k_1}$ and $\beta_2 \in R^{k_2}$ are unknown parameters, X_1 and X_2 are $n \times k_1$ and $n \times k_2$ full column rank matrices respectively, ε is an error term with the elliptically contoured distributions $E_n(0, \sigma^2 V, \psi)$ for $\sigma \in R^+$ and un-structured known matrix $V \in S(n)$. Here $S(n)$ denotes the set of all positive definite matrix of order $(n \times n)$ with the following characteristic function $\phi_\varepsilon(\tau) = \psi(\sigma^2\tau' V\tau)$, for some functions $\psi : [0, \infty] \longrightarrow R$ say characteristic generator (Fang et al. [11]). If ε possess a density, then it can be represented as a scale mixture between a normal distribution with scale parameter t^2 and a weight function $\omega(t)$ by (Chu [8])

$$f_\varepsilon(u) = d_n|\sigma^2 V|^{-\frac{1}{2}} g_n\left(\frac{1}{2\sigma^2}u'V^{-1}u\right) = \int_0^\infty p_N(u|t)\omega(t)dt, \tag{1.2}$$

where d_n is a normalizing constant, $g_n(\cdot)$ is a function called density generator, and $p_N(u|t)$ is the probability density function (pdf) of $N_n(0, \sigma^2 t^{-1} V)$. Then we will write $\varepsilon \sim E_n(0, \sigma^2 V, g_n)$. The condition $\int_0^\infty x^{\frac{n}{2}-1} g_n(x)dx < \infty$ guarantees that $g_n(x)$

* Corresponding author at: School of Science, East China Institute of Technology, Nanchang 330013, China.
E-mail addresses: hgk1997@163.com, huguikai97@163.com (G. Hu), yshh1966@163.com (S. Yu), luoh126@126.com (H. Luo).

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is a density generator. Also g_n and ψ determine each other for each specific member of this family. The mean of ε is the zero-vector and the covariance-matrix of ε is

$$Cov(\varepsilon) = -2\sigma^2\psi'(0)V = \sigma_\varepsilon^2 V, \quad \text{where } \sigma_\varepsilon^2 = -2\sigma^2\psi'(0).$$

The class of ECDs contains some important distributions, such as multivariate normal, multivariate student's t, multivariate Pearson Type VII and so on. For more examples, see Arashi et al. [3]. The elliptically contoured distribution has also some particular mathematical properties and is widely used in statistical inference. For more details, the readers are referred to Cambanis et al. [7], Muirhead [18], Fang et al. [12] and Gupta and Varga [15]. Moreover, ECDs can be regarded as a proper alternative for multivariate normal distribution satisfying robustness and stableness in improved estimation. Such related results have been obtained in different statistical model with elliptically contoured errors. For example, Arashi et al. [3,5] discussed the estimation of parameters in parallelism model and regression model, respectively. Under a balanced loss function, Arashi [1,2] analyzed the problem of estimation in multiple regression model and a seemingly unrelated regression system, respectively.

However, most econometric relationships are subject to specification errors arising from the following three problems: (i) The true functional forms of economic relationships are usually unknown. (ii) Econometric models cannot be specified with omitting some relevant explanatory variables. (iii) Data on economic variables contain measurement errors. Consequently, misspecification of models is difficult to avoid. Our analytical framework is a linear regression model, which is misspecified due to the exclusion of relevant explanatory variable. Such a framework has both theoretical and practical appeals. Technically, the misspecified model allows the inclusion of the correctly specified model as a special case and is thus a more general framework than the latter model. Practically, it is often the case that the deterministic part of the regression cannot be fully specified by the investigator due to the lack of data, ignorance or simplification in the real world. So it makes sense to investigate the properties of parametric estimators in a misspecified regression. In the literature, some conclusions with a normal misspecified linear model have been obtained. For example, for the regression coefficients, Mittelhammer [17] compared the predictive squared error risk behavior of a restricted least squares estimator, preliminary test estimator, and Stein rule estimator. Ohtani [20] examined the dominance of the preliminary test Stein-rule estimator using the Stein-variance estimator over the traditional Stein-rule estimator. Namba [19] obtained the predictive mean squared errors performances of the biased estimators. For the error variance, Ohtani [21] gave the exact distribution of a preliminary test estimator and analyzed theoretically the MSE dominance of the Stein variance estimator over the usual estimator. Ohtani and Wan [23] gave the comparison of the Stein and the usual estimators under the Pitman nearness criterion. However, No systematic work has been done about the estimation of parameters in a misspecified linear regression model with elliptically contoured distribution. In this paper, we concentrate our attention on investigating the properties of error variance estimators.

Now, suppose that the regressor X_2 in the model (1.1) is omitted mistakenly and the model is specified as

$$y = X_1\beta_1 + \eta, \quad \eta = X_2\beta_2 + \varepsilon. \tag{1.3}$$

Such related results have been done about the estimation of error variance in many different linear models. For example, Clarke et al. [9] discussed the preliminary test estimation in a linear regression model with multivariate normal disturbances. Saleh [16] gave the theory of preliminary test and Stein-type estimators and its applications in several kinds of linear model with multivariate normal errors. Giles [14] analyzed the risk performance of preliminary test estimators in a misspecified linear model with spherically symmetric disturbances. Arashi and Tabatabaey [6] gave the improved variance estimation under sub-space restriction and multivariate t errors. Arashi et al. [4] obtained the mathematical characteristics of three improved estimators of variance components in elliptically contoured models. Now, we will propose the least squares estimator, restricted least squares estimator, preliminary test estimator, Stein-type estimator of σ_ε^2 in the misspecified linear model with elliptically contoured errors and give the risk comparison of these estimators by theoretical analysis and numerical evaluations.

The rest of this paper is organized as follows. In Section 2, we give four estimators of σ_ε^2 and obtain the explicit formulas for the risks of proposed estimators. In Section 3, we analyze the risk performances of four estimators in theory, respectively. Meanwhile, we give the dominance picture of risk by numerical evaluation when the error term obeys a multivariate t distribution. In Section 4, we give a bootstrap procedure to estimate the risks of proposed estimators. Concluding remarks are placed in Section 5.

2. Estimators and its risks

In the model (1.3), the generalized least squares estimator of β_1 is

$$\tilde{\beta}_1 = (X_1'V^{-1}X_1)^{-1}X_1'V^{-1}y = C_{11}^{-1}X_1'V^{-1}y, \quad C_{11} = X_1'V^{-1}X_1.$$

Similarly, the least squares estimator (LSE) of σ_ε^2 is

$$S^2 = \frac{(y - X_1\tilde{\beta}_1)'V^{-1}(y - X_1\tilde{\beta}_1)}{m}, \quad m = n - k_1.$$