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Parametric bootstrap approaches for two-way MANOVA with unequal cell sizes and unequal cell covariance matrices☆

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HIGHLIGHTS

- We propose a parametric bootstrap (PB) test in heteroscedastic two-way MANOVA.
- The PB test is invariant under affine- and permutation-transformations.
- The PB test is independent of the choices of weights used to identify the parameters.

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ABSTRACT

In this article, we propose a parametric bootstrap (PB) test for testing main, simple and interaction effects in heteroscedastic two-way MANOVA models under multivariate normality. The PB test is shown to be invariant under permutation-transformations, and affine-transformations, respectively. Moreover, the PB test is independent of the choice of weights used to define the parameters uniquely. The proposed test is compared with existing Lawley–Hotelling trace (LHT) and approximate Hotelling T^2 (AHT) tests by the invariance and the intensive simulations. Simulation results indicate that the PB test performs satisfactorily for various cell sizes and parameter configurations when the homogeneity assumption is seriously violated, and tends to outperform the LHT and AHT tests for moderate or larger samples in terms of power and controlling size. In addition, simulation results also indicate that the PB test does not lose too much power when the homogeneity assumption is actually valid or the model assumptions are approximately correct.

1. Introduction

There has been a continuous interest in checking the significance of the effects of two factors A and B, each having a and b levels respectively, in a two-way multivariate factorial layout. This problem is referred to as two-way multivariate analysis of variance (MANOVA) which is widely used in experimental sciences, e.g., biology, psychology and physics, among others; examples may be found in [8,30] and references therein. When the cell covariance matrices are known to be equal, the available tests such as the classical Wilks likelihood ratio (WLR), Lawley–Hotelling trace (LHT), Bartlett–Nanda–Pillai (BNP) and Roy's largest root tests may be used [1]. These classical MANOVA tests, however, may have serious Type I error problems when the homogeneity of the cell covariance matrices assumption is seriously violated. For example, [30] found that for the nominal size 5%, the empirical size of the LHT test for interaction effect tests could be as large as 75% or as small as 0% in their simulations.

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Recently, there are the following methods proposed for solving this problem. One of the methods is due to [8,9] who attacked this problem via modifying the classical WLR, LHT, and BNP tests, resulting in the so-called modified WLR, LHT, and BNP tests. The large-a asymptotics of these modified MANOVA tests, when the level of one factor tends to infinity, are studied. For finite samples, however, they showed via some simulation studies that these large-a asymptotics are less useful. To overcome this difficulty, [8] proposed to approximate the null distributions of two SSCP (sum of squares and cross-product) matrices involved in the modified MANOVA test statistics by some Wishart distributions with degrees of freedom estimated from the data via matching the mean vectors and total variances; see some details in Section 2.2 of [31]. In view of three main drawbacks of the tests involved in [8], the second method is due to [30] who proposed an approximate Hotelling T^2 (AHT) test. A Wald-type test statistic is used. Its null distribution is approximated by a Hotelling T^2-distribution with one parameter estimated from the data. Some simulation studies conducted in [30] showed that the AHT test outperforms the modified LHT test of [8]. This indicates that the modified MANOVA tests can be further improved. The third method is due to [31] who aimed to show how the modified MANOVA tests in [8] can be improved via estimating the degrees of freedom of the random matrices in the test statistics in a better way and how to make these modified MANOVA tests affine-invariant.

Although the latter two methods indeed improve the modified MANOVA tests, these approaches admit some of the following three main drawbacks: (1) the two methods need to consider the selected weights when the cell sizes are unequal; (2) the associated Wald-type test statistic of AHT is asymmetric in samples; and (3) there are two special cases in which the AHT test may not perform well, see some details in Section 2.6 of [30]. Moreover, for the AHT test, the relationship between the estimated approximate degrees of freedom and the sample cell covariance matrices is very complicated.

For the case of nonnormality and covariance heteroscedasticity assumptions, [21] addressed the problem adapting results presented by Brunner, Dette, and Munk (BDM) [5] and Vallejo and Ato (modified Brown–Forsythe, MBF) [20] in the context of univariate factorial and split-plot designs and a multivariate version of the linear model (MLM) to accommodate heterogeneous data. They compared these procedures with the Welch–James approximate degrees of freedom multivariate statistics based on ordinary least squares via Monte Carlo simulation. The numerical studies show that of the methods evaluated, only the modified versions of the BDM and MBF procedures were robust to violations of underlying assumptions.

In this article, we propose a parametric bootstrap (PB) test for heteroscedastic two-way MANOVA models under multivariate normality. We use standardized effects sum of squares and a natural test statistic obtained by replacing cell covariance matrices by the corresponding sample cell covariance matrices. The PB test admits several nice properties: (1) it can be simply conducted by a routine Monte Carlo algorithm; (2) it is shown to be invariant under permutation-transformations and affine-transformations; (3) The PB test is dependent of choices of the weights used to identify the parameters; and (4) it works well. Simulation results reported in Section 4 indicate that the PB test performs satisfactorily for various cell sizes and parameter configurations when the homogeneity assumption is seriously violated, and tends to outperform the LHT and AHT tests for moderate or larger samples in terms of power and controlling size.

The problem of comparing the mean vectors of k multivariate populations with unequal population covariance matrices is referred to as the heteroscedastic one-way MANOVA which is the most related topic to the heteroscedastic two-way MANOVA. When $k = 2$, this problem is often referred to as the multivariate Behrens–Fisher (BF) problem and it has been well addressed in the literature by various authors including [3,6,14,16–18,28,29], among others. When $k > 2$, the problem of testing equality of the mean vectors in the heteroscedastic one-way MANOVA is more complex, and some approximate solutions are available. These solutions were proposed by [7,12,13], among others. Recently, a PB approach has been proposed by [15]. Our PB solution to the heteroscedastic two-way MANOVA is an essential extension of the solution to the heteroscedastic one-way MANOVA [15]. The main ideas of the proposed PB test are closely related to the work by [27]. In particular, the PB solution for the multivariate case is an extension of our solution to the univariate heteroscedastic two-way ANOVA. Several invariance properties of the PB solution were proved theoretically. This makes the approach a useful contribution for practical applications. Moreover, the robustness of the PB solution to nonnormal data was also investigated. Regarding the parametric bootstrap methodology that we have proposed here, note that the bootstrap can obviously be carried out both parametrically and nonparametrically [22,23]. However, the problems addressed in this paper are in a strict parametric setting, namely the two-way MANOVA with the usual normality assumptions, and heterogeneous cell covariance matrices. Thus we have chosen to do the bootstrap parametrically. The methodologies for the PB tests are presented in Section 2. Proofs of the main results are given in Section 3. Simulation results and an example are presented in Sections 4 and 5 respectively. Finally, some concluding remarks are given in Section 6.

2. Methodologies

2.1. Two-way MANOVA models

Consider a two-way experiment model with factors A and B, with factor levels $A_1, \ldots, A_a$ and $B_1, \ldots, B_b$, respectively, giving a total of ab factorial combinations or treatment cells. Suppose a r-variate random sample of size n_{ij} is available from (i, j)th cell, $i = 1, \ldots, a; j = 1, \ldots, b$. Let $\boldsymbol{Y}_{ijk}, i = 1, \ldots, a; j = 1, \ldots, b; k = 1, \ldots, n_{ij}$ represent these random vectors and $\boldsymbol{y}_{ijk}$ represent their observed (sample) values. Assume that $n_{ij} > r$ so that positive definite sample covariance matrices can be computed for each cell of the design. Suppose that $\boldsymbol{Y}_{ijk}$ satisfy the following model:

$$\boldsymbol{Y}_{ijk} = \boldsymbol{\mu}_{ij} + \boldsymbol{e}_{ijk}, \boldsymbol{e}_{ijk} \sim N_r(\boldsymbol{0}, \boldsymbol{\Sigma}_{ij}), \quad i = 1, \ldots, a; j = 1, \ldots, b;\ k = 1, \ldots, n_{ij}, \tag{2.1}$$