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A mixed model for complete three or higher-way layout with two random effects factors

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ABSTRACT

The classical F-test for testing the hypothesis of no fixed main effects in a mixed model is valid under normality, variance homogeneity and symmetry assumption. We consider a mixed model in which one fixed and two random main effects are crossed. A new test procedure for testing the hypothesis of no fixed main effects is developed under violations of normality, variance homogeneity and symmetry assumptions. The asymptotic distribution of the proposed test statistic is studied under the condition that the numbers of levels of two main random effects are large. The asymptotic distribution of the test statistic is the chi-square distribution. The theory presented in this article is applicable for complete four or higher-way layout with two random factors.

1. Introduction

Consider a design having a factor A with fixed effects and two factors B and C with random effects where A, B and C are crossed. Let Y_{ijkl} denote the lth observation in which A is at the ith level, B at jth and C at lth. Then Y_{ijkl} is expressed as

$$Y_{ijkl} = \mu + \alpha_i^A + a_j^B + a_k^C + a_{ij}^{AB} + a_{ik}^{AB} + a_{jk}^{BC} + a_{ijk}^{ABC} + e_{ijkl} \tag{1}$$
$$i = 1, 2, \ldots, I, \qquad j = 1, 2, \ldots, J, \qquad k = 1, 2, \ldots, K, \qquad l = 1, 2, \ldots, L,$$

where the main and interaction effects are identifiable under the side conditions:

$$\sum_{i=1}^{I} \alpha_i^A = \sum_{i=1}^{I} a_{ij}^{AB} = \sum_{i=1}^{I} a_{ik}^{AB} = \sum_{i=1}^{I} a_{ijk}^{ABC} = 0.$$

The mixed model (1) requires the normality, variance homogeneity and symmetry assumption for the F-test for the hypothesis of no fixed main effects. The normality and variance homogeneity assumption together state that a_j^B is iid $N(0, \sigma_B^2)$, a_k^C is iid $N(0, \sigma_C^2)$, a_{ij}^{AB} is iid $N(0, \sigma_{AB}^2)$, a_{ik}^{AC} is iid $N(0, \sigma_{AC}^2)$, a_{jk}^{BC} is iid $N(0, \sigma_{BC}^2)$, a_{ijk}^{ABC} is iid $N(0, \sigma_{ABC}^2)$ and e_{ijkl} is iid $N(0, \sigma_e^2)$ and the random main and interaction effects are independent from the error terms. The symmetry assumption specifies that the random main and interaction effects are all independent.

In general when any balanced mixed model keeps the side conditions but violates the normality and/or variance homogeneity assumption, the F-test is robust with respect to departure from these assumptions (see Arnold [2], Akritas and Arnold [1] and Bathke [3]). Their results are extended to a multivariate setting by Gupta et al. [5]. However the robustness of the F-test against both normality and homogeneity does not hold in the unbalanced case.

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By assuming that all random main and interaction effects are independent, the mixed model (1) employs seven independent sources of variability (i.e. two random effects factors, four random interaction factors and the error term). Assume that the levels of the random main effects factor B and C are from two independent simple random samplings, then a_j^B, a_{ij}^{AB} and a_{ijk}^{ABC} are dependent as they are all functions of the jth level of B while a_k^C, a_{ik}^{AC} and a_{ijk}^{ABC} are dependent as they are functions of the kth level of C. Then we have only three independent sources of variability (i.e. the selection of the levels of two random effects factors and the error term). This has not received much attention. Recently Gaugler and Akritas [4] have reported consequences of violation of the symmetry assumption.

We establish the mixed model by extending the two-way layout developed by Scheffé [13] and Gaugler and Akritas [4]. The established mixed model is less restrictive than the mixed model (1) since it requires the side conditions but does not disclose the classical assumptions. We develop a new test procedure for the hypothesis of no fixed main effects under the less restrictive model. The proposed test statistic is asymptotically chi-square distributed and a sample value of the test statistic can easily be obtained from data. The asymptotic theory is under the condition that the number of levels of both random effects are large.

This paper is organized as follows: in Section 2 we develop the mixed model without the classical assumptions, in Section 3 the null asymptotic distribution of the proposed test statistic is derived and in Section 4 a consistent estimator of the asymptotic covariance matrix used in calculation of the test statistic is established. Section 5 reports results of simulations comparing the proposed test with the approximate F and Hotelling's T^2 test. Section 6 shows that the theory presented in this article is applicable for the complete higher-way layout with two random factors.

2. The model

Suppose that the levels of the random factors B and C are obtained by two independent random samplings from the populations $\mathcal{T}$ and $\mathcal{V}$ characterized by the distribution P_T and P_V respectively. Let T and V be randomly selected elements from the corresponding population $\mathcal{T}$ and $\mathcal{V}$ where T and V are independent. Then Y_{iTV} is an observation from the combinations of levels (i, T, V) where i denotes a fixed main effects level.

We write Y_{iTV} as

$$Y_{iTV} = E[Y_{iTV}|T, V] + (Y_{iTV} - E[Y_{iTV}|T, V]) = m(i, T, V) + e_{iTV}.$$

The random mean $m(i, T, V)$ is decomposed into the main and interaction effects as

$$m(i, T, V) = \mu + \alpha_i^A + a^B(T) + a^C(V) + a_i^{AB}(T) + a_i^{AC}(V) + a^{BC}(T, V) + a_i^{ABC}(T, V). \tag{2}$$

We shall use the following notation

$$m(\cdot, T, V) = I^{-1}\sum_{i=1}^{I} m(i, T, V), \qquad m(i, \cdot, V) = E_T[m(i, T, V)],$$
$$m(i, T, \cdot) = E_V[m(i, T, V)], \qquad m(i, \cdot, \cdot) = E_{(T,V)}[m(i, T, V)].$$

The decomposition of $m(i, T, V)$ in (2) is unique when we define the main and interaction effects in a conventional way by letting

$$\mu = m(\cdot, \cdot, \cdot), \qquad \alpha_i^A = m(i, \cdot, \cdot) - m(\cdot, \cdot, \cdot), \qquad a^B(T) = m(\cdot, T, \cdot) - m(\cdot, \cdot, \cdot), \tag{3}$$

$$a_i^{AB}(T) = m(i, T, \cdot) - m(i, \cdot, \cdot) - m(\cdot, T, \cdot) + m(\cdot, \cdot, \cdot), \tag{4}$$

$$a^{BC}(T, V) = m(\cdot, T, V) - m(\cdot, T, \cdot) - m(\cdot, \cdot, V) + m(\cdot, \cdot, \cdot), \tag{5}$$

$$a_i^{ABC}(T, V) = m(i, T, V) - m(i, T, \cdot) - m(i, \cdot, V) - m(\cdot, T, V) + m(i, \cdot, \cdot) + m(\cdot, T, \cdot) + m(\cdot, \cdot, V) - m(\cdot, \cdot, \cdot). \tag{6}$$

We have omitted writing $a^C(V)$ and $a^{BC}(T, V)$ since their definitions are similar to those $a^B(T)$ and $a^{AB}(T)$.

The present paper focuses on testing for no fixed main effects of the mixed model (1) without the classical assumption. For this reason we can reduce the random mean $m(i, T, V)$ in (2) as

$$m(i, T, V) = \mu + \alpha_i^A + \beta_i(T) + \omega_i(V) + \gamma_i(T, V) \tag{7}$$

by defining $\beta_i(T)$, $\omega_i(V)$ and $\gamma_i(T, V)$ as

$$\beta_i(T) = a^B(T) + a_i^{AB}(T),$$
$$\omega_i(V) = a^C(V) + a_i^{AC}(V)$$

and

$$\gamma_i(T, V) = a^{BC}(T, V) + a_i^{ABC}(T, V).$$