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The asymptotic behaviors for least square estimation of multi-casting autoregressive processes☆

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ABSTRACT

This paper mainly discusses the asymptotic properties of multi-casting autoregressive processes. By using the m-dependence of random vectors, we prove that the least squares (LS) estimator of the unknown parameters satisfies the moderate deviation principle. Two examples of regular cases are also presented.

1. Introduction

Multi-casting autoregressive (MCAR) process is an adaptation of the autoregressive process to multi-casting tree structured data. It was used to model signal transmission, where each individual in one generation (mother) gives birth to m offsprings (daughters) in the next generation. Fig. 1 shows a 2nd-casting data consisting of 7 observations. Powel's data, G15+C data and cell lineage data are also typically of this kind [6,9]. The initial individual is labeled 1, and m offsprings of individual t are labeled $[\frac{t}{m}]$, $[\frac{t}{m}]+1, \dots, [\frac{t}{m}]+m-1$, where $[u]$ denotes the largest integer no more than u. Let X_t denote the quantitative characteristic of individual t. Then the pth-order MCAR process (MCAR (p)) is given, for all $t \geq 1$, by

$$X_t = \theta_1 X_{[\frac{t}{m}]} + \cdots + \theta_p X_{[\frac{t}{m^p}]} + \epsilon_t. \tag{1}$$

The error sequence $\{\epsilon_t\}$ represents environmental effects while $\theta_1, \theta_2, \dots, \theta_p$ are p unknown real parameters satisfying that all the roots of

$$\phi(z) := 1 - \theta_1 z - \theta_2 z^2 - \cdots - \theta_p z^p = 0 \tag{2}$$

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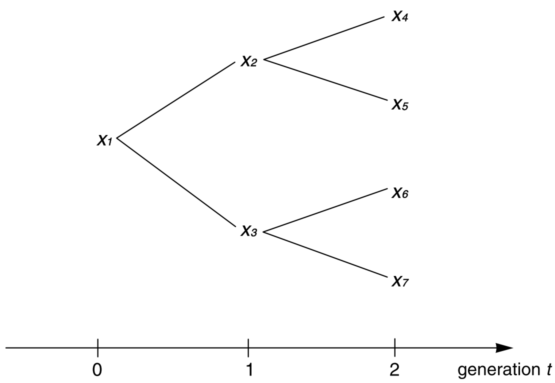

Fig. 1. Multi-casting data with $m = 2$.

are greater than 1 in absolute value. The driven errors $\{(\epsilon_{[\frac{t}{m}]}, \ldots, \epsilon_{[\frac{t}{m}]+m-1}), i = 1, 2, \ldots\}$ are usually supposed to be independent and identically distributed (i.i.d.) with mean zero vector and variance–covariance matrix Ψ given by

$$\Psi = \sigma^2 \begin{pmatrix} 1 & \rho & \ldots & \rho \\ \rho & 1 & \ldots & \rho \\ \vdots & \vdots & \ddots & \vdots \\ \rho & \rho & \ldots & 1 \end{pmatrix}_{m\times m}, \tag{3}$$

where $\sigma^2 > 0$. Obviously, if $m = 2$, MCAR (p) reduces to the pth-order bifurcating autoregressive model. Also, setting $m = 1$ gives the standard AR(p) time series. Several extensions of the model have been proposed. On the one hand, we refer the readers to Hwang and Basawa [12] and Hwang and Baek [11] for statistical inference on branching Markov processes. On the other hand, higher order processes, where not only the effects of the mother but also those of the grand-mother and higher order ancestors are taken into account, have been investigated by Bercu et al. [1]. Recently, an asymmetric model with missing data has been introduced by de Saport et al. [8], where only the effects of the mother are considered, but sister cells are allowed to have different conditional distributions. We can also mention a recent work of Hwang and Basawa [13] dealing with a model of asymmetric multivariate Markov chains on a Galton–Watson tree instead of a regular m-tree.

The purpose of this paper is to investigate the asymptotic properties of the least squares (LS) estimator on the unknown parameters $\theta_1, \theta_2, \ldots, \theta_p$ of general MCAR models. Some results on statistical inference and asymptotic properties of the estimator about these models have been obtained in the literature. The readers can refer to [14] for the central limit theorem (CLT) on the m-regular tree, [12] for CLT on Galton–Watson tree and [8] for asymptotic behaviors of bifurcating autoregressive processes with missing data. But few papers dealt with the exponential convergence rate of the least squares (LS) estimator on the unknown parameters. In this paper, we prove that the LS estimator on the unknown parameter of multi-casting autoregressive processes satisfies the moderate deviation principle (MDP) by using m-dependent random variables (MDP is an intermediate estimation between CLT and the large deviation principle, which is often used to give further estimations related to CLT and the law of iterated logarithm, see [7,17]). Very recently, we noticed that Bitseki Penda and Djellout in [3,4] considered the MDP on the bifurcating autoregressive processes by applying the MDP for martingale, but our model and methods are different from theirs. Their conditions on martingale satisfying MDP cannot be easily checked in our multi-casting autoregressive processes ($m \geq 3$).

2. Preliminaries

In order to state our results, we define the MCAR(p) process $\{X_i, i = 1, 2, \ldots\}$ specified by (1) and (3) again. The method comes from [14]. m is always assumed to be a fixed positive integer in the sequel. MCAR(p) with $m = 1, p = 1$ reduces to the classical linear autoregressive process. In (1), m-variate random errors

$$\left\{\left(\epsilon_{[\frac{t}{m}]}, \ldots, \epsilon_{[\frac{t}{m}+m-2]}, \epsilon_{[\frac{t}{m}]+m-1}\right),\ t = 1, 2, \ldots\right\}$$

are assumed i.i.d. (not necessarily Gaussian) with zero mean vector and variance–covariance matrix Ψ defined in (3). Each individual X_t in one generation produces exactly m-daughters $(X_{[\frac{t}{m}]}, \ldots, X_{\frac{t}{m}+m-2}, X_{[\frac{t}{m}]+m-1})$ in the next generation. To view MCAR(p) as a time series along with the generation, we shall rewrite $\{X_i\}$ in (1) in terms of $\{X_t(j)\}$ for which $X_t(j)$ refers to the jth observation on the tth generation. Here, $t = 0, 1, \ldots$ and $j = 1, \ldots, m^t$. To illustrate, a data structure with $m = 3$ is shown in Fig. 2. To identify the ancestral path of the observation $X_t(j)$, we need the notation

$$\{X_{t-i}(t(j)),\ i = 0, 1, \ldots, t\},$$