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Deflation-based separation of uncorrelated stationary time series

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ABSTRACT

In this paper we assume that the observed p time series are linear combinations of p latent uncorrelated weakly stationary time series. The problem is then to find an estimate for an unmixing matrix that transforms the observed time series back to uncorrelated time series. The so called SOBI (Second Order Blind Identification) estimate aims at a joint diagonalization of the covariance matrix and several autocovariance matrices with varying lags. In this paper, we propose a novel procedure that extracts the latent time series one by one. The limiting distribution of this deflation-based SOBI is found under general conditions, and we show how the results can be used for the comparison of estimates. The exact formula for the limiting covariance matrix of the deflation-based SOBI estimate is given for general multivariate MA(∞) processes. Finally, a whole family of estimates is proposed with the deflation-based SOBI as a special case, and the limiting properties of these estimates are found as well. The theory is widely illustrated by simulation studies.

1. Introduction

The blind source separation (BSS) model is a semiparametric model, where the components of the observed p-variate vector $\boldsymbol{x}$ are assumed to be linear combinations of the components of an unobserved p-variate source vector $\boldsymbol{z}$. The BSS model can then simply be written as $\boldsymbol{x} = \Omega \boldsymbol{z}$, where Ω is an unknown full rank $p \times p$ mixing matrix, and the aim is, based on the observations $\boldsymbol{x}_1, \dots, \boldsymbol{x}_T$, to find an estimate of the mixing matrix Ω (or its inverse). Notice that, in the independent component analysis (ICA), see for example [9], which is perhaps the most popular BSS approach, it is further assumed that the components of $\boldsymbol{z}$ are mutually independent and at most one of them is Gaussian.

It is often assumed in BSS applications that the observation vectors $\boldsymbol{x}_1, \dots, \boldsymbol{x}_T$ are independent and identically distributed (iid) random vectors. However, in this paper $\boldsymbol{x}_1, \dots, \boldsymbol{x}_T$ are observations of a time series. We assume that the p-variate observations $\boldsymbol{x}_1, \dots, \boldsymbol{x}_T$ obey a BSS model such that

$$\boldsymbol{x}_t = \Omega \boldsymbol{z}_t, \quad t = 0, \pm 1, \pm 2, \dots \tag{1}$$

where Ω is a full-rank $p \times p$ mixing matrix, and $\boldsymbol{z} = (\boldsymbol{z}_t)_{t=0,\pm1,\pm2,\dots}$ is a p-variate time series that satisfies

(A1) $E(\boldsymbol{z}_t) = 0$,
(A2) $E(\boldsymbol{z}_t \boldsymbol{z}_t') = I_p$, and

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(A3) $E(\boldsymbol{z}_t\boldsymbol{z}'_{t+k}) = E(\boldsymbol{z}_{t+k}\boldsymbol{z}'_t) = D_k$ is diagonal for all $k = 1, 2, \ldots$.

The p time series in $\boldsymbol{z}$ are thus weakly stationary and uncorrelated. Note that an ICA time series model is obtained if (A3) is replaced by a stronger assumption that the p component series of $\boldsymbol{z}_t$ are mutually independent. No parametric assumptions on the distribution of $\boldsymbol{z}_t$ or on the autocovariance structures are made. For parametric maximum likelihood estimates of the unmixing matrix for Gaussian autoregressive sources with known and unknown covariance structures, see e.g. [7,21].

The aim in model (1) is to find, given $(\boldsymbol{x}_1, \ldots, \boldsymbol{x}_T)$, an estimate $\hat{\Gamma}$ of an unmixing matrix Γ such that $\Gamma\boldsymbol{x}$ has uncorrelated components. $\Gamma = \Omega^{-1}$ is naturally one possible unmixing matrix. Notice that Ω and $\boldsymbol{z}$ in the definition are confounded in the sense that the signs and order of the components of $\boldsymbol{z}$ (and the signs and order of the columns of Ω, respectively) are not uniquely defined.

In the signal processing community, model (1) is often described as a model with components which are temporally correlated but spatially uncorrelated or as a "colored" data model as opposite to the "white" iid model [5,6]. Contrary to the iid ICA model, multiple Gaussian sources are not excluded in the BSS model (1) assumption. In ICA higher order moments are often used to recover the underlying sources, whereas in model (1) the use of second order statistics is adequate, and the separation is based on the information coming from the serial dependence. Due to this property, this blind source separation approach is also called the second order source separation (SOS) approach. BSS time series models have been widely used in engineering, financial, risk management and brain imaging applications, for example. For some recent work with different applications, see e.g. [4,8,11].

Many ICA or SOS methods first whiten the data and then search for an orthogonal matrix to obtain the final solution. This is done by optimizing some objective function, which in some cases is equivalent to jointly diagonalizing certain matrices (see for example [18], for an overview of BSS methods based on joint diagonalization). There are then basically two general ways in which the orthogonal matrix is obtained, either by finding its rows one after another (deflation-based method) or simultaneously (symmetric method). Perhaps the most popular ICA procedure, the so called fastICA, for example has both a deflation-based version and a symmetric version [9].

The outline of this paper is as follows. In Section 2.1 we first discuss the so called AMUSE estimator [19], which was the first estimator of the unmixing matrix developed for this problem, and is based on the simultaneous diagonalization of two autocovariance matrices. The behavior of this estimate depends heavily on the chosen lags and as a solution to this dilemma the so called SOBI estimator [1], which jointly diagonalizes $K > 2$ autocovariance matrices, is considered in Section 2.2. A new deflation-based estimation procedure for a joint diagonalization of autocovariance matrices is proposed. In Section 3 the asymptotical properties of the new SOBI estimate are derived. In Section 4, a large family of unmixing matrix estimates is proposed with the deflation-based SOBI as a special case, and the limiting properties of these estimates are found as well. The limiting behavior and finite-sample behavior of the estimates are illustrated in simulation studies in Section 5. The paper is concluded with some final remarks in Section 6.

2. BSS functionals based on autocovariance matrices

2.1. Functionals based on two autocovariance matrices

Let us first recall the statistical functional corresponding to the AMUSE (Algorithm for Multiple Unknown Signals Extraction) estimator introduced by Tong et al. [19]. Assume that $\boldsymbol{x}$ follows a blind source separation (BSS) model such that, for some lag $k > 0$, the diagonal elements of the autocovariance matrix $E(\boldsymbol{z}_t\boldsymbol{z}'_{t+k}) = D_k$ are distinct, and write

$$S_k = E(\boldsymbol{x}_t\boldsymbol{x}'_{t+k}) = \Omega D_k \Omega', \quad k = 0, 1, 2, \ldots$$

for the autocovariance matrices. The unmixing matrix functional Γ_k is then defined as a $p \times p$ matrix that satisfies

$$\Gamma_k S_0 \Gamma'_k = I_p \quad \text{and} \quad \Gamma_k S_k \Gamma'_k = \Lambda_k,$$

where Λ_k is a diagonal matrix with the diagonal elements in a decreasing order. Notice that Γ_k is affine equivariant, that is, if Γ_k and Γ^*_k are the values of the functional in the BSS model (1) at $\boldsymbol{x}$ and $\boldsymbol{x}^* = A\boldsymbol{x}$ for some non-singular $p \times p$ matrix A, then $\Gamma^*_k = \Gamma_k A^{-1}$ and further $\Gamma_k\boldsymbol{x} = \Gamma^*_k\boldsymbol{x}^*$ (up to sign changes of the components).

The statistical properties of the AMUSE estimator were studied recently in [12]. The exact formula for the limiting covariance matrix was derived for MA(∞) processes, and the asymptotic as well as finite sample behavior was investigated. It was shown that the behavior of the AMUSE estimate depends crucially on the choice of the lag k; the p time series can for example be separated consistently only if the eigenvalues in Λ_k are distinct. Without additional information on the time series, it is not possible to decide which lag k should be used in the estimation. To circumvent this problem [1] proposed the SOBI (Second Order Blind Identification) algorithm seeking an unmixing matrix that makes several autocovariance matrices $S_0, S_1, \ldots, S_K$ as diagonal as possible. In [1], an algorithm based on iterative Jacobi rotations is used to (approximately) jointly diagonalize the autocovariance matrices. In this paper, we propose a new approach in which the latent uncorrelated time series are found in a deflation-based manner (one by one).