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Mixtures of skewed Kalman filters

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ABSTRACT

Normal state-space models are prevalent, but to increase the applicability of the Kalman filter, we propose mixtures of skewed, and extended skewed, Kalman filters. To do so, the closed skew-normal distribution is extended to a scale mixture class of closed skew-normal distributions. Some basic properties are derived and a class of closed skew-t distributions is obtained. Our suggested family of distributions is skewed and has heavy tails too, so it is appropriate for robust analysis. Our proposed special sequential Monte Carlo methods use a random mixture of the closed skew-normal distributions to approximate a target distribution. Hence it is possible to handle skewed and heavy tailed data simultaneously. These methods are illustrated with numerical experiments.

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1. Introduction

State-space models have been widely investigated and used in applied fields such as computer vision, economics, engineering and statistics. The main idea of the state-space model is that the observation y_t at time t is generated by the observation and the state equations. Error terms are usually assumed to follow normal distributions independently.

The assumption of normality in the Kalman filter is not satisfied for a number of real applications. For example, the distributions in a state-space model can be skewed. Inspired by this idea, Naveau et al. [28] proposed a skewed Kalman filter based on the closed skew-normal distribution originally developed by González-Farías et al. [21,22]. We develop a new skewed Kalman filter and an extended skewed Kalman filter, and then extend these models to mixtures of skewed and extended skewed Kalman filters. They include the skewed Kalman filter as a special case when the mixing distribution is degenerated to 1. So we can handle skewed and heavy tailed data simultaneously. Furthermore, from a computational perspective, our extended skewed Kalman filter is faster than the model given by Naveau et al. [28] since there is no need to calculate some mean and covariance terms using numerical techniques.

To implement the skewed Kalman filter we extend the mixture Kalman filter [14] to the mixture of skewed Kalman filters in a direct way. These authors nicely defined partial conditional dynamic linear models and then developed the extended mixture Kalman filter (EMKF). The main idea of EMKF is to extract as many linear and Gaussian components from the system as possible, and then to integrate these components out using the Kalman filter before running a Monte Carlo filter on the

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remaining components. Since, given nonlinear components, the extended skewed Kalman filter becomes linear and with Gaussian errors, the EMKF is directly employed to implement it.

The closed skew-normal distribution [21,22] is the basis distribution for the skewed Kalman filter [28]. It is a family of distributions which includes the (skew-)normal distributions as special cases. Furthermore it contains some skew-normal distributions suggested by Arnold and Beaver [6] and Liseo and Loperfido [25]. It preserves some important properties of the normal distribution, for instance, being closed under: marginalization, conditional distributions, linear transformation (full column or full row rank), sums of independent random variables from this family, and joint distribution of independent random variables in this family; see [17] for an overview.

We extend the closed skew-normal distribution to the scale mixtures of closed skew-normal distributions. Some basic properties are also obtained. As a special case, a class of closed skew-t distributions is derived in explicit form. The scale mixtures of closed skew-normal distributions contain the scale mixtures of skew-normal distributions and the scale mixtures of normal distributions [1,31] as special cases by the aforementioned relationships. The scale mixtures of skew-normal distributions appeared in [12], which include (skew-)normal distributions as special cases. One particular case of this distribution is the skew-normal distribution having a degenerate mixing density. The scale mixtures of normal distributions are widely used. For example, Choy et al. [15] applied those to the study of robust analysis of a normal location parameter, Chen et al. [13] used those for Bayesian modeling of correlated binary responses, and Bradley et al. [11] investigated non-Gaussian state-space models using those, to name a few.

This paper is organized as follows. In Section 2, a scale mixture class of closed skew-normal distributions is derived. Some basic properties are also obtained. As a special case, a class of closed skew-t distributions is derived in explicit form. Using the scale mixtures of closed skew-normal distributions, we develop two different versions of mixtures of skewed Kalman filters in Section 3. In Sections 3.3 and 3.5, we suggest simple generating methods for mixtures of (extended) skewed Kalman filters. We illustrate these methods with numerical experiments in Section 4.

2. A scale mixture class of closed skew-normal distributions

2.1. Definition and some examples

A random vector W has a multivariate closed skew-normal distribution according to González-Farías et al. [21,22] if it has the following probability density function (pdf):

$$C\phi_n(w;\mu,\Sigma)\Phi_m\{D(w-\mu);\nu,\Delta\},\quad w\in\mathbb{R}^n, \tag{1}$$

where $n\geq 1$, $m\geq 1$, $\mu\in\mathbb{R}^n$, $\nu\in\mathbb{R}^m$, $D\in\mathbb{R}^{m\times n}$, $\Sigma\in\mathbb{R}^{n\times n}$ and $\Delta\in\mathbb{R}^{m\times m}$ are both covariance matrices. Here $\phi_n(w;\mu,\Sigma)$ and $\Phi_m(w;\mu,\Sigma)$ are the normal pdf and cumulative distribution function (cdf) with mean μ and covariance matrix Σ. The normalizing constant C of the density function (1) is defined by

$$C^{-1}=\Phi_m(0;\nu,\Delta+D\Sigma D^T). \tag{2}$$

We shall then write $W\sim \mathrm{CSN}_{n,m}(\mu,\Sigma,D,\nu,\Delta)$. When $D=0$, it reduces to the normal distribution. Furthermore when $m=\Delta=1$ and $\nu=0$, it becomes the skew-normal distribution suggested by Azzalini and Dalla Valle [9] and Azzalini and Capitanio [7].

The closed skew-normal distribution will be extended to scale mixtures of closed skew-normal distributions similar to scale mixtures of (skew-)normal distributions. From now on all proofs can be found in the Appendix except for simple cases.

Lemma 1. *Let W and Z be defined as follows:*

$$W=\mu+K(\lambda)^{1/2}\epsilon_1,$$
$$Z=-\nu+D\epsilon_1+\epsilon_2,$$

for given λ where $\epsilon_1\sim N_n(0,\Sigma)$ and $\epsilon_2\sim N_m(0,\Delta)$ are independent random vectors. Here $n\geq 1$, $m\geq 1$, $\mu\in\mathbb{R}^n$, $\nu\in\mathbb{R}^m$, $\Sigma\in\mathbb{R}^{n\times n}$ and $\Delta\in\mathbb{R}^{m\times m}$ are both covariance matrices, $D\in\mathbb{R}^{m\times n}$ is an arbitrary matrix, λ is a mixing variable and $K(\lambda)$ is a weight function. Then the conditional distribution $(W|Z\geq 0,\lambda)\overset{d}{=}V\sim CSN_{n,m}(\mu,K(\lambda)\Sigma,K(\lambda)^{-1/2}D,\nu,\Delta)$.

Hence we define the pdf of scale mixtures of closed skew-normal distributions as follows:

$$f_W(w)=C\int_0^\infty \phi_n\{w;\mu,K(\lambda)\Sigma\}\Phi_m\{K(\lambda)^{-1/2}D(w-\mu);\nu,\Delta\}dH(\lambda),$$

where C is defined in (2) and $H(\lambda)$ is a cdf. We shall then write $W\sim \mathrm{SMCSN}_{n,m}(\mu,K(\lambda)\Sigma,K(\lambda)^{-1/2}D,\nu,\Delta)$. Thus the conditional distribution $W|\lambda\sim \mathrm{CSN}_{n,m}(\mu,K(\lambda)\Sigma,K(\lambda)^{-1/2}D,\nu,\Delta)$. Therefore there is a simple stochastic representation of the above class of distributions:

$$W=\mu+K(\lambda)^{1/2}Y, \tag{3}$$

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