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Robust monitoring of CAPM portfolio betas II

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ABSTRACT

In this work, we extend our study in Chochola et al. [7] and propose some robust sequential procedure for the detection of structural breaks in a *Functional* Capital Asset Pricing Model (FCAPM). The procedure is again based on M-estimates and partial weighted sums of M-residuals and "robustifies" the approach of Aue et al. [3], in which ordinary least squares (OLS) estimates have been used. Similar to Aue et al. [3], and in contrast to Chochola et al. [7], high-frequency data can now also be taken into account. The main results prove some null asymptotics for the suggested test as well as its consistency under local alternatives. In addition to the theoretical results, some conclusions from a small simulation study together with an application to a real data set are presented in order to illustrate the finite sample performance of our monitoring procedure.

1. Introduction and statistical framework

Main aim of this work is to continue and extend our study in Chochola et al. [7] concerning the robust monitoring of CAPM portfolio betas. The Capital Asset Pricing Model (CAPM), introduced by Sharpe [18] and subsequently modified by many authors (see, e.g. Lintner [14], Merton [15] and others), is a still very popular and widely used model for evaluating the risk of a portfolio of assets with respect to the market risk. However, it is also well-known that the pricing of assets and predictions of risks may be incorrect and misleading if the model parameters $\boldsymbol{\beta}_i$ are varying over time. As in Aue et al. [3], we adopt here the arguments of Ghysels [9] and study a (piecewise) unconditional CAPM, rather than a conditional version of the latter (cf., e.g., Andersen et al. [1] for a comprehensive review), since in many cases misspecified conditional CAPMs tend to produce larger pricing errors. For a more extensive discussion of this fact, we refer to Aue et al. [3], Sections 1 and 2, and the references mentioned therein.

Indeed, contributing to avoid pricing and prediction errors was the main motivation for Aue et al. [3] in constructing a sequential monitoring procedure for the testing of the stability of portfolio betas. The corresponding stopping rules in [3] are based on comparing the (ordinary) least squares estimate (OLS) of the beta from a historical data set (training period) to that from sequentially incoming new observations, and they were able to take high-frequency data into account which is a typical situation in nowadays' market analyses (see also Chochola et al. [7] and the references mentioned therein).

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Since OLS estimates may be sensitive with respect to outliers, we tried to "robustify" the Aue et al. [3] approach in [7] by making use of M-estimates instead of least squares estimates and so are able to deal with heavier tail distributions than the OLS procedure. In a first step, however, we confined ourselves there to a study of the CAPM without high-frequency observations. Aim of our present work now is to extend the latter study to the *Functional* Capital Asset Pricing Model (FCAPM) taking also high-frequency observations into account. It will turn out that, even in this more general situation, some moment conditions may be relaxed (cf., e.g., (B.4) below compared to the corresponding assumption in [7]), but that, on the other hand and similar to Aue et al. [3], certain smoothness conditions have to be added concerning the model's intra-day behavior over time (see, e.g., (A.1)–(A.3), (B.5) and (B.7) below).

Note that, via L_p–m-approximability type conditions (cf. (B.4)–(B.5) below), our model is suitable for covering general types of weak dependencies rather than strong dependencies in the sense of long memory. Monitoring procedures in the latter situation are still open for future work. On the other hand, in contrast to [3], our present approach is now applicable to data sets under heavy-tailed (leptocurtic) and contaminated distributions observed at high frequencies, which is certainly more useful in real data applications. The price to pay, however, is that more involved techniques than those used in Chochola et al. [7] are required now and the computational complexity increases as well. Nevertheless, a similar robust sequential monitoring procedure can be constructed for the FCAPM portfolio betas, now also covering a high-frequency situation as described below.

We would like to mention, however, that our focus here is on the methodological and theoretical side, trying to extend the work of Aue et al. [3] by using a robust approach and that of Chochola et al. [7] by including high-frequency situations. Moreover, for the sake of illustration and comparison, we used the same data set as in [3] for our application and a similar setting in the small simulation study of Section 3.

Our statistical framework in the sequel will be as follows. We consider the model

$$\boldsymbol{r}_i(s) = \boldsymbol{\alpha}_i + \boldsymbol{\beta}_i r_{iM}(s) + \boldsymbol{\varepsilon}_i(s), \quad i \in \mathbb{Z},\ s \in [0,1], \tag{1.1}$$

where $\boldsymbol{r}_i(s) = (r_{i,1}(s), \ldots, r_{i,d}(s))^T$ is a d-dimensional vector of (functional) log-returns at (say) "day" i and "intra-day time" s, $r_{iM}(s)$ is the log-return of the market portfolio at day i and time s, and $\boldsymbol{\varepsilon}_i(s) = (\varepsilon_{i,1}(s), \ldots, \varepsilon_{i,d}(s))^T$ are d-dimensional (functional) error terms. The $\boldsymbol{\alpha}_i$'s and $\boldsymbol{\beta}_i$'s are d-dimensional unknown parameters, and the $\boldsymbol{\beta}_i$'s are the parameters of interest, usually called the "portfolio betas". Note that the sequence $\{(\boldsymbol{r}_i(\cdot), r_{iM}(\cdot))\}$ is a $(d+1)$-dimensional (functional) time series satisfying certain conditions to be specified below.

We assume that a training sample of size m with no instabilities is available, i.e.,

$$\boldsymbol{\alpha}_1 = \ldots = \boldsymbol{\alpha}_m =: \boldsymbol{\alpha}_0 = (\alpha_1^0, \ldots, \alpha_d^0)^T, \qquad \boldsymbol{\beta}_1 = \ldots = \boldsymbol{\beta}_m =: \boldsymbol{\beta}_0 = (\beta_1^0, \ldots, \beta_d^0)^T, \tag{1.2}$$

where $\boldsymbol{\alpha}_0$ and $\boldsymbol{\beta}_0$ are unknown parameters. The problem of the instability of the portfolio betas is formulated as a testing problem, that is, we want to test the null hypothesis

$$H_0:\ \boldsymbol{\beta}_1 = \cdots = \boldsymbol{\beta}_m = \boldsymbol{\beta}_{m+1} = \cdots$$

of "no change" versus the alternative

$$H_A:\ \boldsymbol{\beta}_1 = \cdots = \boldsymbol{\beta}_{m+k^*} \neq \boldsymbol{\beta}_{m+k^*+1} = \cdots$$

of a "structural break" at an unknown change-point $k^* = k_m^*$.

For later convenience we reformulate our model as follows:

$$r_{i,j}(s) = \alpha_j^0 + \beta_j^0 r_{iM}(s) + (\alpha_j^1 + \beta_j^1 r_{iM}(s))\delta_m I\{i > m + k^*\} + \varepsilon_{i,j}(s), \quad j = 1, \ldots, d,\ i = 1, 2, \ldots,\ s \in [0,1], \tag{1.3}$$

where $k^* = k_m^*$ is the change-point and $\alpha_j^0, \beta_j^0, \alpha_j^1, \beta_j^1, \delta_m$ are unknown parameters.

As in [7], our test procedures will be generated by convex loss functions $\varrho_1, \ldots, \varrho_d$ with a.s. derivatives $\varrho_j' = \psi_j$ called score functions having further properties to be specified later. The estimators $\widehat{\alpha}_{jm} = \widehat{\alpha}_{jm}(\psi_j)$, $\widehat{\beta}_{jm} = \widehat{\beta}_{jm}(\psi_j)$ of $\alpha_j^0,\ \beta_j^0$ based on the training sample are defined as minimizers of

$$\sum_{i=1}^{m} \sum_{\nu=1}^{n} \varrho_j(r_{i,j}(s_\nu) - a_j - b_j r_{iM}(s_\nu)) \tag{1.4}$$

w.r.t. a_j, b_j, for $j = 1, \ldots, d$, where $s_\nu = \nu/n$, $\nu = 1, \ldots, n$, are n equidistant intra-day time-points.

The test procedure constructed below will be based on functionals of partial sums of weighted M-residuals, which are defined as follows:

$$\boldsymbol{\psi}(\widehat{\boldsymbol{\varepsilon}}_i(s_\nu)) = (\psi_1(\widehat{\varepsilon}_{i,1}(s_\nu)), \ldots, \psi_d(\widehat{\varepsilon}_{i,d}(s_\nu)))^T \tag{1.5}$$

with

$$\widehat{\boldsymbol{\varepsilon}}_i(s_\nu) = (\widehat{\varepsilon}_{i,1}(s_\nu), \ldots, \widehat{\varepsilon}_{i,d}(s_\nu))^T,$$

$$\widehat{\varepsilon}_{i,j}(s_\nu) = r_{i,j}(s_\nu) - \widehat{\alpha}_{jm} - \widehat{\beta}_{jm} r_{iM}(s_\nu). \tag{1.6}$$