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# Detecting changes in cross-sectional dependence in multivariate time series

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ABSTRACT

Classical and more recent tests for detecting distributional changes in multivariate time series often lack power against alternatives that involve changes in the cross-sectional dependence structure. To be able to detect such changes better, a test is introduced based on a recently studied variant of the sequential empirical copula process. In contrast to earlier attempts, ranks are computed with respect to relevant subsamples, with beneficial consequences for the sensitivity of the test. For the computation of $p$-values we propose a multiplier resampling scheme that takes the serial dependence into account. The large-sample theory for the test statistic and the resampling scheme is developed. The finite-sample performance of the procedure is assessed by Monte Carlo simulations. Two case studies involving time series of financial returns are presented as well.

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## 1. Introduction

Given a sequence $\boldsymbol{X}_1, \dots, \boldsymbol{X}_n$ of $d$-dimensional observations, change-point detection aims at testing

$$H_0 : \exists F \text{ such that } \boldsymbol{X}_1, \dots, \boldsymbol{X}_n \text{ have c.d.f. } F \tag{1.1}$$

against alternatives involving the nonconstancy of the c.d.f. Under $H_0$ and the assumption that $\boldsymbol{X}_1, \dots, \boldsymbol{X}_n$ have continuous marginal c.d.f.s $F_1, \dots, F_d$, we have from the work of Sklar [35] that the common multivariate c.d.f. $F$ can be written in a unique way as

$$F(\boldsymbol{x}) = C\{F_1(x_1), \dots, F_d(x_d)\}, \quad \boldsymbol{x} \in \mathbb{R}^d,$$

where the function $C : [0, 1]^d \to [0, 1]$ is a *copula* and can be regarded as capturing the dependence between the components of $\boldsymbol{X}_1, \dots, \boldsymbol{X}_n$. It follows that $H_0$ can be rewritten as $H_{0,m} \cap H_{0,c}$, where

$$H_{0,m} : \exists F_1, \dots, F_d \text{ such that } \boldsymbol{X}_1, \dots, \boldsymbol{X}_n \text{ have marginal c.d.f.s } F_1, \dots, F_d, \tag{1.2}$$

$$H_{0,c} : \exists C \text{ such that } \boldsymbol{X}_1, \dots, \boldsymbol{X}_n \text{ have copula } C. \tag{1.3}$$

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Classical nonparametric tests for $H_0$ are based on sequential empirical processes; see e.g. [2], [11, Section 2.6] and [20]. For moderate sample sizes, however, such tests appear to have little power against alternative hypotheses that leave the margins unchanged but that involve a change in the copula, i.e., when $H_{0,m} \cap (\neg H_{0,c})$ holds. Empirical evidence of the latter fact can be found in [19, Section 4]. For that reason, nonparametric tests for change-point detection particularly sensitive to changes in the dependence structure are of practical interest.

Several tests designed to capture changes in cross-sectional dependence structure were proposed in the literature. Tests based on Kendall's tau were investigated by Gombay and Horváth [16] (see also [17]), Quessy et al. [28] and Dehling et al. [12]. Although these have good power when the copula changes in such a way that Kendall's tau changes as well, they are obviously useless when the copula changes but Kendall's tau does not change or only very little. Tests based on functionals of sequential empirical copula processes were considered in [30,7,38,39]. However, the power of such tests is often disappointing; see Section 5 for some numerical evidence.

It is our aim to construct a new test for $H_0$ that is more powerful than its predecessors against alternatives that involve a change in the copula. The test is based on sequential empirical copula processes as well, but the crucial difference lies in the computation of the ranks. Whereas in [30] and subsequent papers, ranks are always computed with respect to the full sample, we propose to compute the ranks with respect to the relevant subsamples; see Section 2 for details. The intuition is that in this way, the copulas of those subsamples are estimated more accurately, so that differences between copulas of disjoint subsamples are detected more quickly. The phenomenon is akin to the one observed in [15] that the empirical copula, which is based on pseudo-observations, is often a better estimator of a copula than the empirical distribution function based on observations from the copula itself. For another illustration in the context of tail dependence functions, see [3].

The paper is organized as follows. The test statistic is presented in Section 2, and its asymptotic distribution under the null hypothesis is found in Section 3. Next, Section 4 contains a detailed description of the multiplier resampling scheme and its asymptotic validity under the null hypothesis. The results of a large-scale Monte Carlo simulation study are reported in Section 5, and two brief case studies are given in Section 6. Section 7 concludes. Proofs and details regarding the simulation study are deferred to Appendices A–C.

In the rest of the paper, the arrow '$\rightsquigarrow$' denotes weak convergence in the sense of Definition 1.3.3 in [37]. Given a set $T$, let $\ell^\infty(T)$ denote the space of all bounded real-valued functions on $T$ equipped with the uniform metric.

## 2. Test statistic

We now describe our test statistic and highlight the difference with the one in [30,7]. Let $\boldsymbol{X}_1, \ldots, \boldsymbol{X}_n$ be random vectors. For integers $1 \le k \le l \le n$, let $C_{k:l}$ be the empirical copula of the sample $\boldsymbol{X}_k, \ldots, \boldsymbol{X}_l$. Specifically,

$$C_{k:l}(\boldsymbol{u}) = \frac{1}{l-k+1} \sum_{i=k}^{l} \mathbf{1}(\hat{\boldsymbol{U}}_i^{k:l} \le \boldsymbol{u}), \tag{2.1}$$

for $\boldsymbol{u} \in [0, 1]^d$, where

$$\hat{\boldsymbol{U}}_i^{k:l} = \frac{1}{l-k+1} (R_{i1}^{k:l}, \ldots, R_{id}^{k:l}), \quad i \in \{k, \ldots, l\}, \tag{2.2}$$

with $R_{ij}^{k:l} = \sum_{t=k}^{l} \mathbf{1}(X_{tj} \le X_{ij})$ the (maximal) rank of $X_{ij}$ among $X_{kj}, \ldots, X_{lj}$. (Because of serial dependence, there can be ties, even if the marginal distribution is continuous; think for instance of a moving maximum process.) An important point is that the ranks are computed within the subsample $\boldsymbol{X}_k, \ldots, \boldsymbol{X}_l$ and not within the whole sample $\boldsymbol{X}_1, \ldots, \boldsymbol{X}_n$. As we continue, we adopt the convention that $C_{k:l} = 0$ if $k > l$.

Write $\Delta = \{(s, t) \in [0, 1]^2 : s \le t\}$. Let $\lambda_n(s, t) = (\lfloor nt \rfloor - \lfloor ns \rfloor)/n$ for $(s, t) \in \Delta$. Our test statistic is based on the difference process, $\mathbb{D}_n$, defined by

$$\mathbb{D}_n(s, \boldsymbol{u}) = \sqrt{n}\, \lambda_n(0, s)\, \lambda_n(s, 1)\, \{C_{1:\lfloor ns \rfloor}(\boldsymbol{u}) - C_{\lfloor ns \rfloor + 1:n}(\boldsymbol{u})\} \tag{2.3}$$

for $(s, \boldsymbol{u}) \in [0, 1]^{d+1}$. For every $s \in [0, 1]$, it gives a weighted difference between the empirical copulas at $\boldsymbol{u}$ of the first $\lfloor ns \rfloor$ and the last $n - \lfloor ns \rfloor$ points of the sample. Large absolute differences point in the direction of a change in the copula.

To aggregate over $\boldsymbol{u}$, we consider the Cramér–von Mises statistic

$$S_{n,k} = \int_{[0,1]^d} \{\mathbb{D}_n(k/n, \boldsymbol{u})\}^2 \, \mathrm{d}C_{1:n}(\boldsymbol{u}), \quad k \in \{1, \ldots, n-1\}.$$

The test statistic for detecting changes in cross-sectional dependence is then

$$S_n = \max_{1 \le k \le n-1} S_{n,k} = \sup_{s \in [0,1]} \int_{[0,1]^d} \{\mathbb{D}_n(s, \boldsymbol{u})\}^2 \, \mathrm{d}C_{1:n}(\boldsymbol{u}). \tag{2.4}$$

Other aggregating functions can be thought of too, leading for instance to Kolmogorov–Smirnov and Kuiper statistics. In numerical experiments, the resulting tests were found to be less powerful than the one based on the Cramér–von Mises statistic and hence are not considered further in this paper.

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