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Robust monitoring of CAPM portfolio betas

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A B S T R A C T

Some robust sequential procedures for the detection of structural breaks in the Capital Asset Pricing Model (CAPM) are proposed and studied. Most of the existing procedures for this model are based on ordinary least squares (OLS) estimates. Here we propose a class of cumulative sum (CUSUM)-type procedures based on *M*-estimates and partial weighted sums of *M*-residuals. The theoretical results are accompanied by a simulation study that compares the proposed procedures with those based on OLS estimates. An application to a real data set is also presented.

1. Introduction and statistical framework

The Capital Asset Pricing Model (CAPM), introduced by Sharpe [25] and subsequently modified by many authors (see, e.g. Lintner [22], Merton [24] and others), is an important and widely used model for evaluating the risk of a portfolio of assets with respect to the market risk. Despite of some shortcomings pointed out by theoreticians and practitioners as well, the wide-spread use of the CAPM is also well-documented (cf., e.g., the report of Martin and Simin [23]). A main advantage of the model is its simplicity in describing the sensitivity of an asset's risk against the market risk, which is essentially expressed through one parameter, the (so-called) portfolio beta. On the other hand, it is also well-known that the corresponding pricing of a portfolio asset heavily relies on the constancy of the betas over time. Confer, for example, the discussion in Ghysels [13] and recently Caporale [7]. So, it may be of great interest to find out whether portfolio betas change significantly over time or not. The latter was a main motivation in Aue et al. [1] for constructing a sequential monitoring procedure for the testing of the stability of portfolio betas, taking also high-frequency data into account. Along the lines of Chu et al. [9], the corresponding stopping rules of Aue et al. [1] are based on comparing the (ordinary) least squares estimate (OLS) of the beta from a historical data set (training period) to that from sequentially incoming new observations. A structural break (change) in the model is then confirmed when the beta significantly changes, that is, when the newly estimated beta exceeds a critical distance from the historical one.

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However, it is well-known that OLS estimators are sensitive with respect to outliers and deviations from normality assumptions. Concerning the possible application of the CAPM this has led to an extensive discussion and numerous suggestions for "robustifying" the use of beta estimates in the prediction of portfolio risks (confer, e.g., Genton and Ronchetti [12] and Martin and Simin [23] together with the works mentioned therein). Indeed, this has also motivated our present paper in which we propose a robust monitoring procedure for testing the stability of CAPM portfolio betas. In doing so, we try to take into account various aspects of the model which should allow for a broader applicability in practice. First of all we suggest a multivariate approach allowing for dependencies within the portfolio. Second we work with a time series model describing possible dependencies over time, and last but not least our approach is based on (multivariate) M-estimators in order to reduce the sensitivity of the statistical decisions against outliers and non-normality assumptions. For some related work on the M-estimation in linear models with dependent errors confer also Wu [26].

In view of the latter aspects the monitoring procedure proposed below extends other sequential testing procedures for detecting an instability of parameters in regression models when a training sample is available (e.g. Chu et al. [9] or Aue et al. [2]), which are typically based on OLS estimators and related L_2-residuals. Here we shall make use of general M-residuals which to the best of our knowledge have not been applied in this context. Only in case of a univariate linear regression model with independent observations, Koubková [21] already studied some similar robust sequential procedures based on cumulative sum (CUSUM)-type test statistics. We would also like to mention that our procedure can be extended to general multivariate linear regression models or even to functional data setups, but this is beyond the scope of the present work and will be studied elsewhere.

In the sequel our statistical framework will be as follows. We consider the model

$$\mathbf{r}_i = \boldsymbol{\alpha}_i + \boldsymbol{\beta}_i r_{iM} + \boldsymbol{\varepsilon}_i, \quad i \in \mathbb{Z}, \tag{1.1}$$

where $\mathbf{r}_i = (r_{i,1}, \dots, r_{i,d})^T$ is a d-dimensional vector of daily log-returns at time i, r_{iM} is the log-return of the market portfolio at time i, and $\boldsymbol{\varepsilon}_i = (\varepsilon_{i,1}, \dots, \varepsilon_{i,d})^T$ are d-dimensional error terms. The $\boldsymbol{\alpha}_i$'s and $\boldsymbol{\beta}_i$'s are d-dimensional unknown parameters, and the $\boldsymbol{\beta}_i$'s are the parameters of interest, usually called the "portfolio betas". Note that the sequence $\{(\mathbf{r}_i, r_{iM})\}$ is a $(d+1)$-dimensional time series satisfying certain conditions to be specified below.

We assume that a training sample of size m with no instabilities is available, i.e.,

$$\boldsymbol{\alpha}_1 = \cdots = \boldsymbol{\alpha}_m =: \boldsymbol{\alpha}_0, \qquad \boldsymbol{\beta}_1 = \cdots = \boldsymbol{\beta}_m =: \boldsymbol{\beta}_0, \tag{1.2}$$

where $\boldsymbol{\alpha}_0$ and $\boldsymbol{\beta}_0$ are unknown parameters. The problem of the instability of the portfolio betas is formulated as a testing problem, that is, we want to test the null hypothesis

$$H_0 : \ \boldsymbol{\beta}_1 = \cdots = \boldsymbol{\beta}_m = \boldsymbol{\beta}_{m+1} = \cdots$$

of no change versus the alternative

$$H_A : \ \boldsymbol{\beta}_1 = \cdots = \boldsymbol{\beta}_{m+k^*} \neq \boldsymbol{\beta}_{m+k^*+1} = \cdots$$

of a structural break at an unknown change-point $k^* = k_m^*$.

For later convenience we reformulate our model as follows:

$$r_{i,j} = \alpha_j^0 + \beta_j^0 \widetilde{r}_{iM} + (\alpha_j^1 + \beta_j^1 \widetilde{r}_{iM})\delta_m I\{i > m + k^*\} + \varepsilon_{i,j}, \quad j = 1, \dots, d, \ i = 1, 2, \dots, \tag{1.3}$$

where $k^* = k_m^*$ is the change-point, $\alpha_j^0, \beta_j^0, \alpha_j^1, \beta_j^1, \delta_m$ are unknown parameters, and

$$\widetilde{r}_{iM} = r_{iM} - \bar{r}_{mM}, \quad \text{with } \bar{r}_{mM} = \frac{1}{m}\sum_{i=1}^{m} r_{iM}. \tag{1.4}$$

Our test procedures will be generated by convex loss functions $\varrho_1, \dots, \varrho_d$ with a.s. derivatives $\varrho_j' = \psi_j$ called score functions having further properties to be specified later. The estimators $\widehat{\alpha}_{jm} = \widehat{\alpha}_{jm}(\psi_j)$, $\widehat{\beta}_{jm} = \widehat{\beta}_{jm}(\psi_j)$ of $\alpha_j^0, \ \beta_j^0$ based on the training sample are defined as minimizers of

$$\sum_{i=1}^{m} \varrho_j(r_{i,j} - a_j - b_j \widetilde{r}_{iM}) \tag{1.5}$$

w.r.t. a_j, b_j for $j = 1, \dots, d$.

Generally, having $m + k$ observations (the training sample of size m plus k new observations) it would be natural to construct the test procedure via comparing estimators of $\beta_1^0, \dots, \beta_d^0$ based on $\mathbf{r}_1, \dots, \mathbf{r}_m$ and on $\mathbf{r}_{m+1}, \dots, \mathbf{r}_{m+k}$, respectively. This, however, would be computationally quite demanding. Therefore we propose a test procedure based on functionals of partial sums of weighted M-residuals which is asymptotically equivalent.

The M-residuals to be used are defined as follows:

$$\boldsymbol{\psi}(\widehat{\boldsymbol{\varepsilon}}_i) = (\psi_1(\widehat{\varepsilon}_{i,1}), \dots, \psi_d(\widehat{\varepsilon}_{i,d}))^T \tag{1.6}$$