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Monitoring procedure for parameter change in causal time series

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ABSTRACT

We propose a new sequential procedure to detect change in the parameters of a process $X = (X_t)_{t\in\mathbb{Z}}$ belonging to a large class of causal models (such as AR(∞), ARCH(∞), TARCH(∞), or ARMA–GARCH processes). The procedure is based on a difference between the historical parameter estimator and the updated parameter estimator, where both these estimators are quasi-likelihood estimators. Unlike classical recursive fluctuation test, the updated estimator is computed without the historical observations. The asymptotic behavior of the test is studied and the consistency in power as well as an upper bound of the detection delay is obtained. Some simulation results are reported with comparisons to some other existing procedures exhibiting the accuracy of our new procedure. This procedure coupled with retrospective tests is applied to solve off-line multiple breaks detection in the daily closing values of the FTSE 100 stock index.

1. Introduction

In statistical inference, many authors have pointed out the danger of omitting the existence of changes in data. Many papers have been devoted to the problem of test for parameter changes in time series models when all data are available, see for instance [15,19,20,22,1]. These papers consider "retrospective" (off-line) changes *i.e.* changes in parameters when all data are available. Another point of view is to progressively detect change when new data arrive; this is the sequential change-point problem.

This problem can be seen as an engineering process control where new data arrive from an information on an industrial system; see for instance [5]. In this paper, we will follow the paradigm of [10] which considers this problem of "on-line segmentation" as a classical hypothesis testing with a fixed probability of type I error. Chu et al. [10] studied the sequential change in the regression model and pointed out the effects of repeating retrospective tests when new data are observed; this can increase the probability of type I error of the test. They developed two procedures based on cumulative sum (CUSUM) of residuals and recursive parameter fluctuations. Their idea has been generalized and several procedures are now based on this approach. Leisch et al. [23] introduced the generalized fluctuation test based on the recursive moving estimator which contains the test of Chu et al. [10] as a special case. Horváth et al. [16] introduced a residual CUSUM monitoring procedure where the recursive parameter is based on the historical data. This procedure has been generalized by Aue et al. [2] to the class of linear models with dependent errors. Berkes et al. [6] considered sequential changes in the parameters of the GARCH process. According to the fact that the functional limit theorem assumed by Chu et al. [10] is not satisfied by the squares of

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residuals of the GARCH process, they developed a procedure based on quasi-likelihood scores. Kirch [21] and Hušková and Kirch [17] pointed out that the critical value of the monitoring procedure is usually based on an infinite observation period whereas it is finite in practice. Such procedure leads to a loss of some power. They proposed some bootstrapping methods to obtain critical values (even for a small sample sizes situation) for sequential change-point tests for linear regression models. The monitoring change in linear models has also been carried out by Hušková et al. [18], Černíková et al. [8]. Na et al. [24] developed a monitoring procedure for the detection of parameter changes in general time series models. They show that under the null hypothesis of no change, their detector statistic weakly converges to a known distribution. However, the asymptotic behavior of this detector is unknown under the alternative of parameter changes.

In this new contribution, we consider a large class of causal time series and investigate the asymptotic behavior under the null hypothesis of no change but also under the alternative hypothesis of change. More precisely, let $M, f : \mathbb{R}^{\mathbb{N}} \to \mathbb{R}$ be measurable functions, $(\xi_t)_{t\in\mathbf{Z}}$ be a sequence of centered independent and identically distributed (iid) random variables satisfying $\mathrm{var}(\xi_0) = \sigma^2$. We assume that the functions M and f are known up to some unknown parameter θ belonging to a fixed compact set $\Theta \subset \mathbb{R}^d$. Let $\mathcal{T} \subset \mathbf{Z}$, and for any $\theta \in \Theta$, define

Class $\mathcal{M}_{\mathcal{T}}(M_\theta, f_\theta)$: *The process $X = (X_t)_{t\in\mathbf{Z}}$ belongs to $\mathcal{M}_{\mathcal{T}}(M_\theta, f_\theta)$ if it satisfies the relation:*

$$X_{t+1} = M_\theta\big((X_{t-i})_{i\in\mathbb{N}}\big)\xi_t + f_\theta\big((X_{t-i})_{i\in\mathbb{N}}\big) \quad \text{for all } t \in \mathcal{T}. \tag{1}$$

The existence and properties of this general class of causal and affine processes were studied in Bardet and Wintenberger [3]. Numerous classical time series (such as AR(∞), ARCH(∞), TARCH(∞), ARMA–GARCH or bilinear processes) can be written as a model $\mathcal{M}_{\mathbf{Z}}(M, f)$. The off-line change detection for such class of models has already been studied in Bardet et al. [4] and Kengne [20].

Suppose now that we observed available historical data $X_1, \ldots, X_n$ with $(X_1, \ldots, X_n) \in \mathcal{M}_{\{1,\ldots,n\}}(M_{\theta_0^*}, f_{\theta_0^*})$ and $\theta_0^* \in \Theta$ is unknown. Then, we observe new data $X_{n+1}, X_{n+2}, \ldots, X_k, \ldots$: the monitoring scheme starts. For each new observation, we would like to know if a change occurs in the parameter θ_0^*. More precisely, we consider the following test problem:

$\mathbf{H}_0$: θ_0^* is constant over the observation $X_1, \ldots, X_n, X_{n+1}, \ldots$ *i.e.* $(X_n)_{n\in\mathbb{N}} \in \mathcal{M}_{\mathbb{N}}(M_{\theta_0^*}, f_{\theta_0^*})$;

$\mathbf{H}_1$: there exist $k^* > n$, $(\theta_0^*, \theta_1^*) \in \Theta^2$, with $\theta_0^* \neq \theta_1^*$, such that $(X_1, \ldots, X_{k^*}) \in \mathcal{M}_{\{1,\ldots,k^*\}}(M_{\theta_0^*}, f_{\theta_0^*})$ and $(X_{k^*+n})_{n\in\mathbb{N}} \in \mathcal{M}_{\{k^*+1,\ldots\}}(M_{\theta_1^*}, f_{\theta_1^*})$.

The main contribution of this paper is a new procedure proposed to test H_0 against H_1. For any $k \geq 1$, $\ell, \ell' \in \{1, \ldots, k\}$ (with $\ell \leq \ell'$) let $\widehat{\theta}(X_\ell, \ldots, X_{\ell'})$ be the quasi-maximum likelihood estimator (QMLE in the sequel) of the parameter computed on $\{\ell, \ldots, \ell'\}$ as it is defined in (7). When new data arrive at time $k > n$, we explore the segment $\{\ell, \ell + 1, \ldots, k\}$ with $\ell \in \{n - v_n, \ldots, k - v_n\}$ (where $(v_n)_{n\in\mathbb{N}}$ is a fixed sequence of integer numbers) that a distance between $\widehat{\theta}(X_\ell, \ldots, X_k)$ and $\widehat{\theta}(X_1, \ldots, X_n)$ is the largest. We construct a detector taking into account a distance between $\widehat{\theta}(X_\ell, \ldots, X_k)$ and $\widehat{\theta}(X_1, \ldots, X_n)$ and if this distance is larger than a suitable critical value, then H_0 is rejected and a model with a new parameter is considered; otherwise, the monitoring scheme continues. We show that this detector is almost surely consistent under H_0 and almost surely diverges to infinity under H_1: the consistency of our procedure follows.

Finally, Monte-Carlo experiments have been done in several scenarios, comparing our procedure to the ones of Horváth et al. [16] (see also Aue et al. [2]) and Na et al. [24]. It appears that our procedure outperforms these other procedures in terms of test power and detection delay in different frames. An application to financial data (FTSE 100 stock index) allows to detect changes in these data in accordance with historical and economic events.

In Section 2 the assumptions and the definition of the quasi-likelihood estimator are provided. In Section 3 we present the monitoring procedure and the asymptotic results. Section 4 is devoted to a simulation study for AR(1) and GARCH(1, 1) processes. In Section 5 we apply our procedure to famous financial data. The proofs of the main results are provided in Section 6.

2. Assumptions and definition of the quasi-likelihood estimator

2.1. Assumptions on the class of models $\mathcal{M}_{\mathbf{Z}}(f_\theta, M_\theta)$

We begin by giving assumptions ensuring the existence and stationarity of a process belonging to a class $\mathcal{M}_{\mathbf{Z}}(f_\theta, M_\theta)$. First, define $\theta \in \mathbb{R}^d$ and

- M_θ and f_θ are numerical functions such that for all $(x_i)_{i\in\mathbb{N}} \in \mathbb{R}^{\mathbb{N}}$, $M_\theta\big((x_i)_{i\in\mathbb{N}}\big) \neq 0$.
- $h_\theta := M_\theta^2$.

We will use the following classical notations:

1. $\|\cdot\|$ applied to a vector denotes the Euclidean norm of the vector;
2. for any compact set $\mathcal{K} \subseteq \mathbb{R}^d$ and for any $g : \mathcal{K} \longrightarrow \mathbb{R}^{d'}$, $\|g\|_{\mathcal{K}} = \sup_{\theta\in\mathcal{K}}(\|g(\theta)\|)$;
3. for any set $\mathcal{K} \subseteq \mathbb{R}^d$, $\overset{\circ}{\mathcal{K}}$ denotes the interior of $\mathcal{K}$.