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On the test for the homogeneity of a parameter matrix with some rows constrained by synchronized order restrictions

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ABSTRACT

The tests on the homogeneity of the columns of the coefficient matrix in a multiple multivariate linear regression with some rows of the matrix constrained by synchronized orderings, using the test statistics obtained by replacing the unknown variance–covariance matrix with its estimator in likelihood ratio test statistics, form a family of ad hoc tests. It is shown in this paper that the tests in the family share the same alpha-level critical values and follow the same distributions for computing their p-values. A sufficient condition is established for other tests to enjoy these properties, and to be more powerful. Two such more powerful tests are examined.

1. Introduction

Let $\beta \in R^{p\times q}$ and positive definite $\Sigma \in R^{p\times p}$ $(\Sigma > 0)$ be the coefficient matrix and the variance–covariance matrix of a multivariate multiple linear regression model. Suppose that the elements of some of the rows of β are constrained by synchronized orderings, the monotone non-decreasing or monotone non-increasing orderings with respect to a common partial order of column indices. For this model we consider the test of the hypothesis that the columns of β are all equal using an ad hoc test statistic obtained by replacing the unknown Σ in a likelihood ratio test (LRT) statistic with its estimator. The row indices of the restricted rows of β form a non-empty set $H \subset \{1, \ldots, p\}$. With all possible H, a family of ad hoc tests is produced.

It is shown in this paper that all members of the ad hoc test family share the same α-level critical values, and a unique distribution can be employed for computing the p-values for all members of the family. A sufficient condition is established for other tests to enjoy the above two properties, and to be more powerful. Two such more powerful tests are examined.

The study of the tests on the homogeneity of multiple parameters with an alternative specifying an ordering dates back to the late 50s. Bartholomew [2] derived an LRT on the homogeneity of independent normal means against a simple ordering. Since then researchers have studied many cases of hypothesis testing under constraints. Mukerjee and Tu [5] investigated the inferences for a polynomial regression with the regression function restricted to be monotonic. For the testing on the restricted components of a multivariate normal mean vector Silvapulle [9] proposed a Hotelling's T^2-type test statistic by replacing the unknown variance–covariance matrix with its estimator. Many results of the statistical inferences on order restricted parameters are summarized in [1,6,10].

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Sasabuchi et al. [8] extend the test in [2] to a test on vector homogeneity against vector simple ordering in which the relation of $\mu_i \le \mu_j$ for vectors μ_i and μ_j is interpreted as "componentwise less than or equal to". The model restriction in [8] is later generalized by Hu [4] from componentwise simple ordering to a general vector quasi ordering. Recently, Sasabuchi [7] explored a more powerful test than that in [8]. The work in this paper is the continuation of the research in this direction, and extended and generalized the results in [7,4].

In the next section we give preliminaries on the problem and explain the notations. Two lemmas and two theorems are presented in Section 3. The proofs of the lemmas, however, are placed in the appendices. In Section 4 we give the main results on the tests. These results are directly based on the conclusions of the two theorems in Section 3.

2. Preliminaries

2.1. Order restrictions

Let $\ll_{(i)}$ be a partial order in $\Omega = \{1, \ldots, q\}$, and $\mathcal{C}_{(i)}$ be either $\{\nu \in R^q : \nu_s \le \nu_t \text{ for } s \ll_{(i)} t\}$ or $\{\nu \in R^q : \nu_s \ge \nu_t \text{ for } s \ll_{(i)} t\}$, $i = 1, \ldots, p$. The rows matrix $(\beta_{(1)}, \ldots, \beta_{(p)})' \in R^{p\times q}$ are under separate order restrictions if $\beta_{(i)} \in \mathcal{C}_{(i)}$ for all $i = 1, \ldots, p$. These separate restrictions are said to be synchronized if $\ll_{(i)}$, $i = 1, \ldots, p$, are identical. Synchronized order restrictions are essentially isotonic restrictions and antitonic restrictions with respect to a common partial order $\ll$. For this $\ll$, let $\mathcal{C}_+$ and $\mathcal{C}_-$ be the collections of all isotonic and antitonic vectors, and $\mathcal{C}_{(i)}$ be either $\mathcal{C}_+$ or $\mathcal{C}_-$. With H, a non-empty subset of $\{1, \ldots, p\}$,

$$\mathcal{C}_H = \{(\beta_{(1)}, \ldots, \beta_{(p)})' \in R^{p\times q} : \beta_{(i)} \in \mathcal{C}_{(i)} \text{ for all } i \in H\} \tag{1}$$

is the collection of all matrices with some rows, the rows specified by H, constrained by synchronized orderings. Here $\mathcal{C}_+$ and $\mathcal{C}_-$ are convex cones in R^q, and $\mathcal{C}_H$ is a convex cone in $R^{p\times q}$. When $H = \{i\}$, the notation $\mathcal{C}_H$ is simplified as $\mathcal{C}_i$. Suppose that H is partitioned by H_+ and H_- such that $\mathcal{C}_{(i)} = \mathcal{C}_+$ for $i \in H_+$, and $\mathcal{C}_{(i)} = \mathcal{C}_-$ for $i \in H_-$. For $f, g \in R^p$ define $f \preceq g$ if $f_i \le g_i$ for all $i \in H_+$ and $f_i \ge g_i$ for all $i \in H_-$. Then $\preceq$ is a quasi order for vectors in R^p, and

$$\mathcal{C}_H = \{(\beta_{(1)}, \ldots, \beta_{(p)})' \in R^{p\times q} : \beta_{(i)} \preceq \beta_{(j)} \text{ for all } i \ll j\}.$$

Thus the restriction $\beta \in \mathcal{C}_H$ is a special case of multivariate isotonic restriction.

Restrictions $\beta \in \mathcal{C}_H$ are frequently encountered in statistics. For example, suppose a survey is conducted among the students in 4th and 5th grades in districts I and II. The means of the age, the household income, the height and the time for non-academic activities (NAA) in school are listed in matrix $\beta = (\beta_{ij})_{4\times 4} = (\beta_{(1)}, \ldots, \beta_{(4)})' \in R^{4\times 4}$.

	4th grade District I	5th grade District I	4th grade District II	5th grade District II
Age	β_{11}	β_{12}	β_{13}	β_{14}
Household income	β_{21}	β_{22}	β_{23}	β_{24}
Height	β_{31}	β_{32}	β_{33}	β_{34}
Time for NAA	β_{41}	β_{42}	β_{43}	β_{44}

It is reasonable to assume that $\beta_{11} \le \beta_{12}$, $\beta_{11} \le \beta_{14}$, $\beta_{13} \le \beta_{12}$ and $\beta_{13} \le \beta_{14}$. Define $\ll$ in $\Omega = \{1, 2, 3, 4\}$ by $1 \ll 2$, $1 \ll 4$, $3 \ll 2$ and $3 \ll 4$, and let $\mathcal{C}_+$ be $\{\nu \in R^4 : \nu_s \le \nu_t \text{ for } s \ll t\}$. The assumption becomes $\beta_{(1)} \in \mathcal{C}_+$. Since the height and the age are positively correlated, the time for non-academic activities and the age are negatively correlated, and the household income and the age are uncorrelated, we may also impose the restrictions $\beta_{(3)} \in \mathcal{C}_+$ and $\beta_{(4)} \in \mathcal{C}_-$ while no restriction is imposed on $\beta_{(2)}$. Thus the restriction is expressed as $\beta \in \mathcal{C}_H$ where $H = \{1, 3, 4\}$.

One can change the signs of some of the variables to make synchronized orderings identical. But to avoid variables such as negative age, negative income, or negative time for non-academic activities, we kept two opposite cone structures for the restrictions in this paper, which did not create any difficulties in mathematical processing.

2.2. A multivariate model

Let the columns of data matrix $Y = (Y_1, \ldots, Y_n) \in R^{p\times n}$ be p-variate independent normal vectors with a common variance–covariance matrix Σ. The distribution of Y is denoted by $Y \sim N_{p\times n}(E(Y), \Sigma)$. This notation, proposed by Hu [3], has many convenient properties. For example, with $A \in R^{q\times p}$ and $B \in R^{q\times n}$,

$$Y \sim N_{p\times n}(E(Y), \Sigma) \Rightarrow AY + B \sim N_{q\times n}(AE(Y) + B, A\Sigma A').$$

The structure of $E(Y)$ is dictated by the specifications of the model for data. For the example described in Section 2.1, suppose data matrix $Y \in R^{4\times 1000}$ is obtained from the observations on the four variables, the age, the household income, the height and the non-academic time in school, of 1000 students. According to multivariate analysis of variance (MANOVA),

$$Y \sim N_{4\times 1000}(\beta X', \Sigma), \qquad X = \begin{pmatrix} 1_{n_1} & 0 & 0 & 0 \\ 0 & 1_{n_2} & 0 & 0 \\ 0 & 0 & 1_{n_3} & 0 \\ 0 & 0 & 0 & 1_{n_4} \end{pmatrix} \quad \text{and} \quad \sum_{i=1}^{4} n_i = 1000.$$