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Subsampling tests for variance changes in the presence of autoregressive parameter shifts

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ABSTRACT

In this paper, we consider the problem of testing for variance changes in the linear autoregressive processes including AR(p) processes when there are autoregressive parameter shifts. In performing a test, we employ the conventional residual CUSUM of squares test (RCUSQ) statistic. The RCUSQ test is based on the subsampling method introduced by Jach and Kokoszka (2004) [16] to eliminate the influence caused by autoregressive parameter shifts. It is shown that under regularity conditions, the test statistic behaves asymptotically the function of a standard Brownian bridge. We establish the asymptotic validity of this method and assess its performance both theoretically and numerically.

1. Introduction

The presence of change-points in key macroeconomics and finance in developed economies appears to be relatively common because a myriad of political and economic factors can cause the relationships among economic variables to change over time. Since the early work of Page [23], Chow [10], Quandt [27] and Brown et al. [9], numerous studies have been undertaken with an upsurge of interest in various models with an unknown change-point. With respect to the problem of testing for structural breaks, recent contributions include [7,1,2,20,19,21,17,18] as well as the monograph by Csörgő and Horváth [11]. Issues about the distributional properties of the estimates, in particular those of break-date, have also been considered by Bai [4,3]. These tests and inference issues have been addressed in the context of multiple structural breaks by Bai and Perron [5].

Owing to the works of Perron [24,25] and Hendry and Neale [15], it is now well recognized in the literature that unit root tests should be designed to have power against the alternative hypothesis that allows for a break in the mean. The conventional unit tests that ignore the break under the alternative can spuriously fail to reject the unit root null hypothesis. Therefore, Perron [25] proposed a unit root test that is specifically designed to have power against the alternative that allows for a one time break in the mean, occurring at a known break-date.

In this article, special attention is paid to the ergodic stationary processes including linear autoregressive (LAR) time series since they accommodate important linear time series models, such as AR(p), which have been central to the analysis of data

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with linear characteristics. For references, see [14,12,22] and the papers cited therein. For AR(p) models, there are $p+2$ parameters, the mean, the variance of the white noise and the p autoregressive parameters. A change in any of these over time is a sign of disturbance that is important to detect. Gombay [13] used maximum likelihood function to test for changes in any one of these $p+2$ parameters separately, or in any collection of them. Unfortunately, he did not consider the influence caused by autoregressive parameter shifts, if we just want to test for variance changes. Since the residual subsampling based test conventionally discards correlation effects and enhances the performance of the test, the goal of this paper is to illustrate that the RCUSQ test based on subsampling methodology of [16] can be used to analyze the autoregressive models where the variance of white noise exhibits a change, while the autoregressive parameter shifts occur.

The organization of this paper is as follows. In Section 2, we present the regular conditions under which the RCUSQ test statistic based on subsampling converges weakly to the function of a standard Brownian bridge. In Section 3, as an illustration we consider the variance changes problem in the presence of autoregressive parameter shifts. Simulation results related to AR(1) process and a empirical application are reported in Section 4. We provide brief concluding remarks in Section 5.

2. Assumptions and models

We consider the following models:

$$\begin{aligned} y_t &= \mu + \xi_t, \\ \xi_t &= \alpha_1\xi_{t-1} + \alpha_2\xi_{t-2} + \cdots + \alpha_p\xi_{t-p} + \varepsilon_t, \end{aligned} \tag{1}$$

where p is a finite positive integer. Assume $\theta = (\alpha_1, \ldots, \alpha_p)$ and the innovations processes ε_t satisfy $E\varepsilon_t = 0$ and $\mathrm{Var}(\varepsilon_t) = \sigma_1^2$.

We test the null hypothesis

$$H_0 : y_1, \ldots, y_T \quad \text{is a sample for some } \sigma_1,$$

against the variance changes alternative under autoregressive parameter shifts

$$H_1 : \exists\, \sigma_1,\ \sigma_2 \quad \text{satisfying } \sigma_1 \neq \sigma_2,\ (\theta \neq \theta^*)$$

where $\theta^* = (\alpha_1^*, \ldots, \alpha_p^*)$. Such that the sample $y_1, \ldots, y_T$ has the form

$$\begin{aligned} y_t &= \mu + \xi_t, \\ \xi_t &= \begin{cases} \alpha_1\xi_{t-1} + \alpha_2\xi_{t-2} + \cdots + \alpha_p\xi_{t-p} + \varepsilon_t, & t \le k^*; \\ \alpha_1^*\xi_{t-1} + \alpha_2^*\xi_{t-2} + \cdots + \alpha_p^*\xi_{t-p} + \varepsilon_t^*, & t > k^*, \end{cases} \end{aligned} \tag{2}$$

where the innovations post the change-point satisfy $E\varepsilon_t^* = 0$ and $\mathrm{Var}(\varepsilon_t^*) = \sigma_2^2$. We assume that $k^* = [T\tau^*]$, $0 < \tau^* < 1$ is a fixed and known break-date.

We state the assumptions which are needed to prove asymptotic validity of our approach.

Assumption 2.1. The independent identical distribution (i.i.d.) innovations ε_t satisfy $E|\varepsilon_t|^{4+\delta} < \infty$, where $\delta > 0$.

Assumption 2.2. All of the roots of $1 - \alpha_1 z - \alpha_2 z^2 - \cdots - \alpha_p z^p = 0$ lie out of the unit circle.

Remark 2.1. The last assumption can ensure $\xi_t = \sum_{j=0}^{\infty} \varphi_j \varepsilon_{t-j}$. Beveridge and Nelson [6] decomposed that $T^{-1}\sum_{t=1}^{T} \xi_t = \varphi(1) \cdot T^{-1}\sum_{t=1}^{T} \varepsilon_t + o_p(1)$, where $\varphi(1) = \sum_{j=0}^{\infty} \varphi_j < \infty$. This shows that the rate of convergence for $T^{-1}\sum_{t=1}^{T} \xi_t$ and $T^{-1}\sum_{t=1}^{T} \varepsilon_t$ are the same.

Our approach also relies on the following results.

Lemma 2.1. *If Assumption 2.1 holds, then*

$$T^{1/2}(\hat{\theta} - \theta) \quad \textit{has a proper, nondegenerate limiting distribution},$$

where $\hat{\theta}$ is least squares estimators.

Remark 2.2. The results can be obtained from [8] and indicate that $|\hat{\alpha}_i - \alpha_i| = O_p(T^{-1/2})$. Since ε_t are not observable, our test is based on residuals $\hat{\varepsilon}_t$ instead of ε_t, which are obtained via estimating the unknown autoregressive parameters.