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Random block matrices generalizing the classical Jacobi and Laguerre ensembles

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ABSTRACT

In this paper we consider random block matrices which generalize the classical Laguerre ensemble and the Jacobi ensemble. We show that the random eigenvalues of the matrices can be uniformly approximated by the zeros of matrix orthogonal polynomials and obtain a rate for the maximum difference between the eigenvalues and the zeros. This relation between the random block matrices and matrix orthogonal polynomials allows a derivation of the asymptotic spectral distribution of the matrices.

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1. Introduction

The three classical ensembles of random matrix theory are the Hermite, Laguerre and Jacobi ensembles. Associated with each ensemble there is a real positive parameter β which is usually considered for three values. The case $\beta = 1$ corresponds to real matrices, while the ensembles for $\beta = 2$ and $\beta = 4$ arise from complex and quaternion random matrices, respectively, according to Dyson [1] threefold classification. Dumitriu and Edelman [2] provided tridiagonal random matrix models for the general β-Hermite and β-Laguerre ensembles for all $\beta > 0$. The development of a tridiagonal matrix model corresponding to the general β-Jacobi ensemble for all $\beta > 0$ was an open problem, which was recently considered by Killip and Nenciu [3]. The spectral distributions of large dimensional matrices of the three classical ensembles have been studied extensively in the literature, see [4,5] or [6].

Several authors have extended the study of random matrices to the case of random block matrices and we refer to the works of Girko [7] and Oraby [8,9], among others. Recently, Dette and Reuther [10] addressed random block matrices which generalize the tridiagonal model of the Hermite ensemble constructed by Dumitriu and Edelman [2] and obtained the asymptotic spectral distribution. It is the purpose of the present paper to investigate the asymptotic properties of some random block tridiagonal matrices corresponding to the classical Laguerre and Jacobi ensembles. In Section 2 we revisit the Jacobi ensemble and introduce the random block matrices considered in this paper. In Section 3 we will review some facts on matrix orthogonal polynomials and the limiting distribution of their zeros. In Section 4 we demonstrate that the eigenvalues of the random block matrices can be approximated uniformly (almost surely) by the deterministic zeros of matrix orthogonal polynomials. Matrix polynomials have been studied by several authors, see [11–19]. In particular, Duran [15] and Dette and Reuther [10] provided limit theorems for the empirical distribution of the zeros of matrix orthogonal polynomials and these results are applied to obtain the asymptotic spectral distribution of the random block matrices. In Section 5 we introduce a generalization of the Laguerre ensemble to block matrices and study the corresponding limiting spectral distribution. Finally, the proofs of some technical results are deferred to an Appendix.

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2. The Jacobi ensemble and the corresponding block matrices

The Jacobi ensemble is defined by its density

$$f_{\beta,\gamma,\delta}(\lambda) = c_{\beta,\gamma,\delta} \prod_{1\le i<j\le n} |\lambda_i - \lambda_j|^\beta \prod_{j=1}^{n} (2-\lambda_j)^\gamma (2+\lambda_j)^\delta I_{(-2,2)}(\lambda_j) \tag{2.1}$$

of the unordered eigenvalues, where $\beta > 0,\ \gamma, \delta > -1$, $I_{(-2,2)}$ denotes the indicator function of the interval $(-2, 2)$ and the normalization constant $c_{\beta,\gamma,\delta}$ is given by

$$c_{\beta,\gamma,\delta} = 4^{-n\left(\gamma+\delta+\frac{n-1}{2}\beta+1\right)} \prod_{j=0}^{n-1} \frac{\Gamma\left(1+\frac{\beta}{2}\right)\Gamma\left(\gamma+\delta+(n+j-1)\frac{\beta}{2}+2\right)}{\Gamma\left(1+\frac{\beta}{2}+\frac{\beta}{2}j\right)\Gamma\left(\gamma+\frac{\beta}{2}j+1\right)\Gamma\left(\delta+\frac{\beta}{2}j+1\right)}.$$

Killip and Nenciu [3] provided a tridiagonal random matrix model of the Jacobi ensemble where the entries are composed of independent random variables with beta distributions on the interval $[-1, 1]$. To be precise, note that the Beta distribution $Beta(a, b)$ on the interval $[-1, 1]$ is defined by the density

$$f(x) = 2^{1-a-b} \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} (1-x)^{a-1}(1+x)^{b-1} I_{(-1,1)}(x). \tag{2.2}$$

Now let α_k for $k = 0, \ldots, 2n-1$ be independent random variables with

$$\alpha_k \sim \begin{cases} Beta\left(\frac{2n-k-2}{4}\beta+\gamma+1,\ \frac{2n-k-2}{4}\beta+\delta+1\right) & \text{for } k \text{ even,} \\ Beta\left(\frac{2n-k-3}{4}\beta+\gamma+\delta+2,\ \frac{2n-k-1}{4}\beta\right) & \text{for } k \text{ odd,} \end{cases}$$

then the joint density of the eigenvalues of the tridiagonal matrix

$$J_n = J_n(\beta,\gamma,\delta) = \begin{pmatrix} b_1 & a_1 & & \\ a_1 & b_2 & \ddots & \\ & \ddots & \ddots & a_{n-1} \\ & & a_{n-1} & b_n \end{pmatrix} \in \mathbb{R}^{n\times n} \tag{2.3}$$

with entries

$$b_{k+1} = (1-\alpha_{2k-1})\alpha_{2k} - (1+\alpha_{2k-1})\alpha_{2k-2},$$
$$a_{k+1} = \left\{(1-\alpha_{2k-1})(1-\alpha_{2k}^2)(1+\alpha_{2k+1})\right\}^{1/2}$$

$(\alpha_{2n-1} = \alpha_{-1} = \alpha_{-2} = -1)$ is given by the Jacobi ensemble (2.1). As a generalization of the matrix J_n, we consider random tridiagonal block matrices of the form

$$J_n^{(p)} = J_n^{(p)}(\beta,\gamma,\delta) := \begin{pmatrix} B_{0,n}^{(p)} & A_{1,n}^{(p)} & & & & \\ A_{1,n}^{(p)} & B_{1,n}^{(p)} & A_{2,n}^{(p)} & & & \\ & A_{2,n}^{(p)} & \ddots & \ddots & \ddots & \\ & & \ddots & \ddots & \ddots & A_{\frac{n}{p}-1,n}^{(p)} \\ & & & & A_{\frac{n}{p}-1,n}^{(p)} & B_{\frac{n}{p}-1,n}^{(p)} \end{pmatrix} \in \mathbb{R}^{n\times n}, \tag{2.4}$$

where $n = mp$ with $m, p \in \mathbb{N}$ and the symmetric $p \times p$ blocks $A_{i,n}^{(p)}$ and $B_{i,n}^{(p)}$ are defined by

$$B_{i,n}^{(p)} := \begin{pmatrix} b_{ip+1}^{(1,n)} & a_{ip+1}^{(1,n)} & a_{ip+2}^{(2,n)} & \cdots & \cdots & a_{(i+1)p-1}^{(p-1,n)} \\ a_{ip+1}^{(1,n)} & b_{ip+2}^{(1,n)} & a_{ip+2}^{(1,n)} & & \cdots\cdots & a_{(i+1)p-1}^{(p-2,n)} \\ a_{ip+2}^{(2,n)} & \vdots & \vdots & \ddots & & \vdots \\ \vdots & & & & b_{(i+1)p-1}^{(1,n)} & a_{(i+1)p-1}^{(1,n)} \\ a_{(i+1)p-1}^{(p-1,n)} & \cdots & \cdots & \cdots & a_{(i+1)p-1}^{(1,n)} & b_{(i+1)p}^{(1,n)} \end{pmatrix}$$

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