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A bivariate Lévy process with negative binomial and gamma marginals

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Abstract

The joint distribution of X and N, where N has a geometric distribution and X is the sum of N IID exponential variables (independent of N), is infinitely divisible. This leads to a bivariate Lévy process $\{(X(t), N(t)), t \geq 0\}$, whose coordinates are correlated negative binomial and gamma processes. We derive basic properties of this process, including its covariance structure, representations, and stochastic self-similarity. We examine the joint distribution of $(X(t), N(t))$ at a fixed time t, along with the marginal and conditional distributions, joint integral transforms, moments, infinite divisibility, and stability with respect to random summation. We also discuss maximum likelihood estimation and simulation for this model.

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1. Introduction

Kozubows and Panorska [11] studied a bivariate distribution with exponential and geometric marginals (BEG), denoted by $\mathcal{BEG}(\beta, p)$ and defined via the stochastic representation

$$(X, N) \stackrel{d}{=} \left(\sum_{i=1}^{N} X_i, N \right). \tag{1}$$

Here, the $\{X_i\}$ are IID exponential variables with the PDF

$$f(x) = \beta e^{-\beta x}, \quad x > 0, \tag{2}$$

and N is a geometric random variable with the PDF

$$h(n) = \mathbb{P}(N = n) = p(1 - p)^{n-1}, \quad n = 1, 2, \ldots, \tag{3}$$

independent of the $\{X_i\}$. They derived basic properties of these models, including marginal and conditional distributions, joint integral transforms, infinite divisibility, stability with respect to geometric summation, and maximum likelihood estimation of the parameters. Applications of the BEG model range from finance and actuarial science to hydrology and climate. In the financial application presented in [11], the process $\{X_i\}$ represented log-returns of the exchange rates between two currencies for their growth/decline periods. The BEG model then provided the joint distribution for the cumulative log-returns and the length of their growth/decline periods. The fit of the BEG model to this data was quite remarkable in terms of marginal as well and bivariate distributions. Similarly, BEG model yields itself useful in actuarial problems of modeling total size of the claims exceeding a certain threshold when the number of claims is geometric.

In hydrology and climate research, the problem of estimating total stream flow, precipitation, Palmer Drought Severity Index, or Pacific Decadal Oscillation index exceeding a threshold (e.g. long term mean or high percentile) is of primary importance to water resource managers and safe engineering design needing reasonable flood, drought, or water storage estimates (see, e.g., [2,6,14,17,18]). These problems are usually studied in terms of *episodes*, represented as a random vector of magnitude and duration, where the duration N is the number of time intervals (e.g. years) the process remains continuously above (or below) a reference level, while the magnitude X is the sum of all process values for a given duration. Here, the BEG model provides a stochastic framework for the joint distribution of (X, N), and has been successfully applied to stream flow analysis in [3].

While the BEG model proved to be quite useful, in many applications a more general model is needed. One important extension arises as the sum of n independent BEG random vectors, whose distribution is the same as the marginal distribution at $t = n$ of a bivariate Lévy process $\{(X(t), N(t)), t \geq 0\}$, where $(X(1), N(1))$ is given by (1). This process, which was briefly mentioned in [11], admits a stochastic representation

$$\{(X(t), N(t)),\ t \geq 0\} \stackrel{d}{=} \left\{ \left(\sum_{i=1}^{NB(t)} X_i + G(t), NB(t) + t \right),\ t \geq 0 \right\}, \tag{4}$$

where the $\{X_i\}$ are as before, $\{G(t),\ t \geq 0\}$ is a gamma Lévy process starting at zero, based on the exponential distribution (2), and $\{NB(t), t \geq 0\}$ is a negative binomial (NB) Lévy process