Journal of Multivariate Analysis 104 (2012) 101–114

Contents lists available at SciVerse ScienceDirect

Journal of Multivariate Analysis

journal homepage: www.elsevier.com/locate/jmva

A note on testing hypotheses for stationary processes in the frequency domain

Holger Dette *, Thimo Hildebrandt

Ruhr-Universität Bochum, Fakultät für Mathematik, 44780 Bochum, Germany

ARTICLE INFO

Article history:
Received 26 April 2010
Available online 29 July 2011

AMS subject classification:
62M10
62M15
62G10

Keywords:
Stationary process
Goodness-of-fit tests
Kernel estimate
Smoothed periodogram
Weak convergence under the alternative

ABSTRACT

In a recent paper, Eichler (2008) [11] considered a class of non- and semiparametric hypotheses in multivariate stationary processes, which are characterized by a functional of the spectral density matrix. The corresponding statistics are obtained using kernel estimates for the spectral distribution and are asymptotically normally distributed under the null hypothesis and local alternatives. In this paper, we derive the asymptotic properties of these test statistics under fixed alternatives. In particular, we also show weak convergence but with a different rate compared to the null hypothesis. We also discuss potential statistical applications of the asymptotic theory by means of a small simulation study.

1. Introduction

In this paper, we investigate hypotheses about the second order properties of a multivariate d-dimensional stationary time series $\{\mathbf{X}_t\}_{t\in\mathbb{Z}}$, which can be expressed in terms of functionals of its spectral density matrix $f = (f_{ij})_{i,j=1,\dots,d}$. This problem has been investigated by numerous authors replacing the unknown density in the functional by a corresponding nonparametric estimate (see [18,19,14] or [8,7] among others). Recently, Eichler [11] proposed a test for a very large class of hypotheses of the form

$$H_0 : \int_{-\pi}^{\pi} \|\Psi(f(\lambda),\lambda)\|^2 \mathrm{d}\lambda = 0 \quad \text{vs.} \quad H_1 : \int_{-\pi}^{\pi} \|\Psi(f(\lambda),\lambda)\|^2 \mathrm{d}\lambda \neq 0, \tag{1.1}$$

where $\Psi : \mathbb{C}^{d\times d} \times [-\pi,\pi] \to \mathbb{C}^r$ is a functional characterizing the null hypothesis by the property $\Psi(f(\lambda),\lambda) \equiv 0$ a.e. on $\Pi = [-\pi,\pi]$ and $\|\cdot\|$ denotes the Euclidean norm on $\mathbb{C}^r$. Typical examples include

$$\Psi(f(\lambda),\lambda) = \left(\frac{f_{11}(\lambda)}{\frac{1}{d}\sum_{i=1}^{m} f_{ii}(\lambda)} - 1, \dots, \frac{f_{dd}(\lambda)}{\frac{1}{d}\sum_{i=1}^{m} f_{ii}(\lambda)} - 1 \right) \tag{1.2}$$

corresponding to the comparison of the diagonal elements or the null hypothesis $H_0 : f_{11} = \cdots = f_{dd}$, and the problem of testing if the components $\mathbf{X}_t^A$ and $\mathbf{X}_t^B$ of the series $\{\mathbf{X}_t\}_{t\in\mathbb{Z}} = \{(\mathbf{X}_t^A, \mathbf{X}_t^B)\}_{t\in\mathbb{Z}}$ are uncorrelated, which corresponds to the

* Corresponding author.
E-mail addresses: holger.dette@ruhr-uni-bochum.de, holger.dette@rub.de (H. Dette).

0047-259X/$ – see front matter
doi:10.1016/j.jmva.2011.07.002

spectral coherence

$$\Psi(f(\lambda),\lambda) = \left(\frac{f_{ij}(\lambda)}{\sqrt{f_{ii}(\lambda)f_{jj}(\lambda)}}\right)_{\substack{i\in\{1,\ldots,d_1\}\\ j\in\{d_1+1,\ldots,d\}}}. \tag{1.3}$$

Eichler [11] proposed to estimate the spectral density matrix f by a kernel estimate, say $\hat{f}$ and showed weak convergence of an appropriately standardized version of the statistic

$$S_T(\Psi) = \int_{-\pi}^{\pi} \|\Psi(\hat{f}(\lambda),\lambda)\|^2 \mathrm{d}\lambda \tag{1.4}$$

under the null hypothesis and local alternatives.

The purpose of the present paper is to provide some more insight in the asymptotic properties of the statistic $S_T(\Psi)$. Eichler [11] proved convergence in probability of $S_T(\Psi)$. For a special case of $S_T(\Psi)$ corresponding to the comparison of two spectral densities Dette and Paparoditis [7] showed weak convergence under fixed alternatives. In the present paper we study the asymptotic properties of the statistic $S_T(\Psi)$ for the general class of functionals Ψ investigated by Eichler [11] in this situation. In particular we will show weak convergence of a standardized version of $S_T(\Psi)$ to a normal distribution with a different rate of convergence compared to the null hypothesis. These results can be used for the construction of confidence intervals for the measure

$$\int_{-\pi}^{\pi} \|\Psi(f(\lambda),\lambda)\|^2 \mathrm{d}\lambda$$

and for testing precise hypothesis regarding this quantity (see [1]). In Section 2 we briefly review the approach of Eichler [11] and state the necessary assumptions for our results, which are given in Section 3. In Section 4 we present two examples illustrating our approach, and investigate the finite sample properties of two statistical applications. Finally, all technical details are deferred to an Appendix.

2. Preliminaries

Let $\{\mathbf{X}_t\}_{t\in\mathbb{Z}}$ denote a centered d-dimensional stationary process. For a matrix A define $A^* = \overline{A}^T$ as the complex conjugated and transposed matrix A. Let

$$I(\lambda) = (2\pi T)^{-1} d(\lambda) d^*(\lambda) \tag{2.1}$$

$$d(\lambda) = \sum_{t=1}^{T} \mathbf{X_t} \mathrm{e}^{-\mathrm{i}\lambda t} \tag{2.2}$$

denote the periodogram. Then the spectral density matrix f can be estimated by

$$\hat{f}(\lambda) = \frac{2\pi}{T} \sum_k K_b(\lambda - \lambda_k) I(\lambda_k), \tag{2.3}$$

where K denotes a kernel function, $K_b(\lambda) = K(\lambda/b)/b$, b is a bandwidth converging to 0 with increasing sample size and $\lambda_k = 2\pi k/T$ ($k = -\lfloor (n-1)/2 \rfloor, \ldots, \lfloor n/2 \rfloor$) denote the Fourier frequencies. Following Eichler [11] we assume that the bandwidth satisfies $b \sim T^{-\nu}$ for some $2/9 < \nu < 1/2$ and that the kernel K is a symmetric, bounded and Lipschitz continuous density. Finally, we assume that the function Ψ is defined on $D \times [-\pi, \pi]$, where $D \subset \mathbb{C}^{d\times d}$ is an open set containing the set $\{f(\lambda) \mid \lambda \in [-\pi, \pi]\}$. Throughout this paper we use the notation $\Pi = [-\pi, \pi]$ and require the following assumptions for our asymptotic results:

(i) $\Psi(Z,\lambda)$ is holomorphic with respect to the variable Z.

(ii) $\Psi(Z,\lambda)$ and its first derivative with respect $z = \mathrm{vec}(Z)$

$$D_z(\Psi(Z,\lambda)) = \frac{\partial \Psi(Z,\lambda)}{\partial z^T}$$

are piecewise Lipschitz continuous with respect to $\lambda \in \Pi = [-\pi, \pi]$.

(iii) There exists a constant ρ such that the closed ball

$$B_{\rho,\lambda} = \{Z \in \mathbb{C}^{d\times d} : \|f(\lambda) - Z\| \le \rho\}$$

is contained in D for all $\lambda \in \Pi$, and such that

$$\sup_{\lambda\in\Pi} \sup_{Z\in B_{\rho,\lambda}} \|\Psi(Z,\lambda)\| < \infty.$$

(iv) $0 < \int_{\Pi} \|D_z(\Psi(f(\lambda),\lambda))\| \mathrm{d}\lambda < \infty$.