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Random change on a Lie group and mean glaucomatous projective shape change detection from stereo pair images

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ABSTRACT

In this article we develop a nonparametric methodology for estimating the mean change for matched samples on a Lie group. We then notice that for $k \geq 5$, a manifold of projective shapes of k-ads in 3D has the structure of a $3k - 15$ dimensional Lie group that is equivariantly embedded in a Euclidean space, therefore testing for mean change amounts to a one sample test for extrinsic means on this Lie group. The Lie group technique leads to a large sample and a nonparametric bootstrap test for one population extrinsic mean on a projective shape space, as recently developed by Patrangenaru, Liu and Sughatadasa. On the other hand, in the absence of occlusions, the 3D projective shape of a spatial k-ad can be recovered from a stereo pair of images, thus allowing one to test for mean glaucomatous 3D projective shape change detection from standard stereo pair eye images.

1. Introduction

Statistical analysis on Lie groups was first considered by Beran [4] in the context of testing for uniformity of a distribution on a compact homogeneous space. Nonparametric density estimation on Lie groups via deconvolution like techniques and Fourier analysis was developed by Kim [28,29], Healy et al. [24], Lesosky et al. [34] and Koo and Kim [32]; these have applications in medical imaging, robotics, and polymer science (see [47,48,33]).

Statistical inference on certain Lie groups in a parametric setting was considered also in concrete estimation problems in paleomagnetics, plate tectonics (see [12–14,16,30,44]), biomechanics [42,44,43], image registration, spatial statistics, etc., using the idea of statistical Lie group model as shown by Chang [15].

Correlation of two images of the same object [16], comparison of mean axes [5], or of extrinsic mean planar projective shapes [35], as well as estimation of 3D motion in computer vision (e.g., [46]), leads also to natural inference problems on Lie groups. However, most of the time the dimension of the Lie group is higher than the dimension of the sampling manifold on which the Lie group is acting. This increase in dimensionality requires usually even larger samples; however this kind of data analysis is relevant in problems in medical imaging, proteomics, face analysis and the like where the sample sizes are often moderate or small. Ideally the dimensions of the Lie group and the sampling manifold where the group is acting should be the same, as motivated by the discussion in Section 3. This is the case if the sampling manifold has a Lie group structure in itself.

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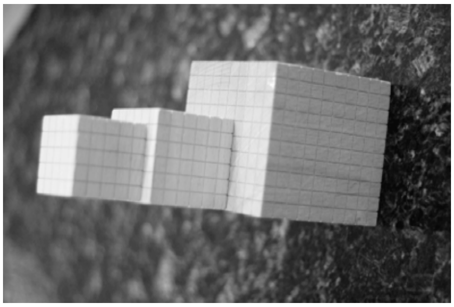

Fig. 1. Image acquisition of planar scenes does not preserve parallelism—see the edges of one of the squares.

In Section 2 we consider such an example of sampling manifold, that is crucial in pattern recognition and medical imaging, the space $P\Sigma_3^k = (\mathbb{R}P^3)^{k-5}$ of projective shapes of 3D configurations of k-ads in general position, $k \geq 5$, for which any four of the first five points are not coplanar, thus defining a *projective frame* [38]. There, in addition, we use the fact that the quaternion multiplication on the unit sphere $S^3 \subset \mathbb{R}^4$ induces a natural group multiplication on $(\mathbb{R}P^3)^{k-5}$.

Following [17], in Section 3 we define the change C in a matched pair of random objects X, Y on a Lie group $(\mathcal{G}, \circ)$ by $C = X^{-1} \circ Y$. This allows us to reduce a two matched pairs problem for data on $\mathcal{G}$, to a one sample mean change test on $\mathcal{G}$.

In Section 4, we briefly review extrinsic means and their asymptotic distributions [8,9] and in particular we apply the nonparametric inference for testing for one sample extrinsic mean on the projective shape manifold $P\Sigma_m^k = (\mathbb{R}P^m)^{k-m-2}$ from [40] to mean projective shape change in 3D.

In Section 5 we describe and display our data. Stereo data of the eye is the most common imaging data for eye disease detection and control. Our stereo pairs are from the Louisiana Experimental Glaucoma Study (LEGS) in Rhesus monkeys [10]. In each of the individuals in the study, an increased internal ocular pressure (IOP) was induced in one eye, while the other eye was left as control. Both eyes were stereo imaged, thus for each individual in the study, a complete paired observation consists in four optic nerve head (ONH) images, two of the control eye (A) and two of the treated eye (B). Our data consists in fifteen independent complete paired observations from LEGS.

Section 6 is dedicated to an application of mean glaucomatous projective shape change detection, from stereo data of the ONH. Since glaucoma is a disease affecting the 3D appearance of the ONH region due to high IOP, it leads to a change in the 3D similarity shape of this region. The group of direct similarities is a subgroup of the group of projective transformations, therefore a projective shape change in the ONH implies a direct similarity shape change as well. After a brief review of 3D projective shape reconstruction from pairs of 2D images, from [40], our focus is on extrinsic mean 3D projective shape change detection due to increased IOP in the eye, using the methodology described in Section 4. The results are statistically significant.

2. Projective shape spaces of 3D configurations as Lie groups

Human vision and pinhole camera image acquisition are based on a central projection principle. Therefore, in pixel coordinates, two images of the same planar scene, acquired by an ideal pinhole digital camera differ by a projective transformation of $\mathbb{R}^2$. Formally, a projective transformation $v = f_A(u)$ of $\mathbb{R}^m$ is a given by the equation

$$v^j = \frac{a^j_{m+1} + \sum_{i=1}^{m} a^j_i u^i}{a^{m+1}_{m+1} + \sum_{i=1}^{m} a^{m+1}_i u^i}, \quad \forall j = 1, \ldots, m \tag{2.1}$$

where $A = ((a^j_i)_{i,j=1,\ldots,m+1})$, is nonsingular $\det(A) \neq 0$. Although similarities, that were previously considered in shape analysis are a very particular case of projective transformations, unlike similarities, projective transformations do not preserve parallelism in general, as seen in Fig. 1, rendering similarity shape analysis from digital images often obsolete.

Two configurations of points in $\mathbb{R}^m$ have the same *projective shape* if they differ by a projective transformation of $\mathbb{R}^m$. Note that in general the maximal domain of definition of a projective transformation is $\mathbb{R}^m$ with a hyperplane removed, therefore projective transformations are acting only on subsets of $\mathbb{R}^m$, and do not have a group structure under composition. For this reason, rather than considering projective shapes of configurations in $\mathbb{R}^m$, we consider projective shapes of configurations in the projective space $\mathbb{R}P^m = (\mathbb{R}^{m+1} \setminus \mathbf{0})/_\sim$, where $x \sim y \iff \exists\lambda \neq 0, x = \lambda y$. The equivalence class under $\sim$ of $x \in \mathbb{R}^{m+1} \setminus \mathbf{0}$ is a *projective point* $[x] \in \mathbb{R}P^m$, and $(x^1, \ldots, x^{m+1}) \in \mathbb{R}^{m+1} \setminus \mathbf{0}$, are called *homogeneous coordinates* of the projective point $[(x^1, \ldots, x^{m+1})]$, which are unique up to a nonzero multiple. Note that since $\mathbb{R}^m$ can be identified with an open affine subset of $\mathbb{R}P^m$, any configuration in $\mathbb{R}^m$ can be regarded as a configuration in $\mathbb{R}P^m$ and the *pseudogroup action* by projective transformations on open dense subsets of $\mathbb{R}^m$ is extended to a group action of the projective group $PGL(m)$