Journal of Statistical Planning and Inference 175 (2016) 1–10

Contents lists available at ScienceDirect

Journal of Statistical Planning and Inference

journal homepage: www.elsevier.com/locate/jspi

Informed traders' hedging with news arrivals

Sang-Hyeon Park [a,*], Kiseop Lee [b]

[a] Daishin Securities, 16, Gugjegeumyung-ro 8-gil, Yeongdeungpo-gu, Seoul 07330, Republic of Korea

[b] Department of Mathematics, University of Louisville, Louisville, KY 40292, USA

ARTICLE INFO

Article history:
Received 6 September 2015
Received in revised form 25 November 2015
Accepted 28 November 2015
Available online 6 January 2016

MSC:
primary 91G20
secondary 60G99

Keywords:
Information asymmetry
Hedging
Incomplete market
Local risk minimization

ABSTRACT

We study a hedging and pricing problem of a market with jumps, where both jump sizes and the timing are affected by exclusive information available only to informed traders. The exclusive information process is a continuous time stochastic process, but affects the price process only at discrete times through jumps. This model is an extension of Lee and Song (2007), where the exclusive information affects only the jump timing, and Kang and Lee (forthcoming), where the exclusive information affects only jump sizes. We find the local risk minimization hedging strategy of informed traders.

1. Introduction

In a market, there are traders with different levels of information, and their strategies heavily depend on the amount of information they have. Naturally, this information asymmetry problem has been a popular research topic in finance for a long time, but more sophisticated mathematical studies on this topic came out just recently. Mathematically, information asymmetry is modeled by the size and the inclusiveness of filtrations. An informed trader who has more information or better interpretation of a public information is assumed to have a larger filtration than those who have less information. Amendinger (2000); Biagini and Oksendal (2005); Corcuera et al. (2004); Föllmer et al. (1999); Hu and Oksendal (2007); Imkeller et al. (2001) adopted this enlargement of filtration as their main modeling tool.

The next issue is how to model the new exclusive information available only to informed traders. The simplest case is when the information is just a single random variable, which will be known to the general public someday. Amendinger (2000); Imkeller et al. (2001); Biagini and Oksendal (2005) fall to this category where the exclusive information that informed traders have is described by a fixed σ-field, i.e. $F = G \vee \sigma(L)$ for some fixed random variable L.

Lee and Song (2007) extended this idea to a general case where the exclusive information comes as a continuous time stochastic process. Under their model, the exclusive information affects jumps of the price process, and the timing of jumps follows a doubly stochastic Poisson process which depends on the information process. While this was a meaningful first step in the sense that the information effect was included in the model, it is more realistic to assume the information has more effect to jump sizes rather than jump times.

* Corresponding author.
E-mail addresses: lanieanb@daishin.com, laniean@yonsei.ac.kr (S.-H. Park), kiseop.lee@louisville.edu (K. Lee).

http://dx.doi.org/10.1016/j.jspi.2015.11.006
0378-3758/

Recently, Kang and Lee (2014) studied the exact case where the exclusive information affects jump sizes of the asset price. Therefore, informed traders know the direction and the strength of the exclusive news.

In this study, we combine models of Lee and Song (2007); Kang and Lee (2014), and study the model where the information affects both the jump timing and the jump size. In other words, we suggest a double dependency jump model. Under this model, we study a hedging strategy of contingent claim. The extension from previous models to this new one is not trivial, since we encounter dependency between jump sizes and jump timing. For instance, finding a compensator of the jump process is challenging. We derive a suitable formula for the compensated measure of a jump process which is related to the information process. We extend the result of Lee and Song (2007), which considered a random measure involved in the jump timing only to the case where both the jump timing and the size are affected by the information process. This model is more complicated than a model with two information processes, which affect the jump timing and jump sizes separately. When one information process affects both the jump timing and jump sizes, it is difficult to find the exact random measure and the corresponding compensator. In our study, we can find the optimal hedging strategy even when we do not know the exact form of those, as long as we have information on jump sizes.

Using the compensated measure of a double dependency jump process, we derive a local risk minimization strategy in Föllmer and Schweizer (1991); Schweizer (1995) as a hedging strategy of contingent claim. The local risk minimization strategy of this model consists of the weighted delta and additional sensitivities associated with jumps and information process. Moreover, we present a numerical example of this model and compare it to that of an uninformed trader. We provide a hedging error in the sense of $E[(C_T - C_0)^2]$ where C_t is a cost process. A sample of the hedging strategy with respect to an informed trader and an uninformed trader is also presented.

The rest of this paper is organized as follows. We introduce the model in Section 2. The minimal martingale measure, which plays an important role to find the optimal hedging strategy, is studied in Section 3. Section 4 gives us the local risk minimization strategies. Numerical examples are provided in Section 5. We conclude in Section 6.

2. Model

We consider a probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{0\le t\le T}, \mathbf{P})$, where $\mathbf{P}$ is an empirical probability measure, and filtration $(\mathcal{F}_t)_{0\le t\le T}$ satisfies the usual conditions. We consider a market which consists of a risky asset and a money market account. There is no dividend and the spot rate of interest is assumed to be zero. All precesses are $\mathcal{F}_t$-adapted. A portfolio (ξ, η) is a vector process, where ξ_t is the number of the underlying asset at time t, and η_t is the amount of the risk less asset at t. Then, the value of a portfolio at t is

$$V_t = \xi_t\, S_t + \eta_t. \tag{2.1}$$

We assume that the explicit information process follows a standard diffusion process

$$dX_t = \alpha(X_t)dt + \beta(X_t)dW_t^X \quad \text{with } X_0 = 0, \tag{2.2}$$

where W^X is a standard Brownian motion. The price of the underlying asset follows the dynamics:

$$dS_t = \mu_0 S_{t-}dt + \sigma S_{t-}dW_t + S_{t-}dR_t \quad \text{with } S_0 = s, \tag{2.3}$$

where W is another standard Brownian motion with $[W, W^X]_t = \rho t$ and $\mu_0, \sigma > 0$. The jump process R_t is defined by

$$\sum_{0<s\le t} U(X_t)1_{\Delta R_s(X_s)\neq 0},$$

where $U(\cdot)$ is a random function which denotes jump sizes.

We assume that $E[U(\cdot)|X_t] := \kappa_1(X_t) \in \mathbb{R}$ and $E[U^2(\cdot)|X_t] := \kappa_2^2(X_t) > 0$. By applying the random measure theory in Jacod and Shiryaev (2003), the jump process R_t can be written as

$$R_t = \int_0^t \int_{-\infty}^{\infty} y\, p^R(X_s, dy, ds),$$

where $p^R(X_t, dy, dt)$ is the associated random measure on $\mathbb{R}\times[0, T]$ for fixed X_t. We assume that there exists a compensated measure $m_1(X_t, dy, dt)$ such that

$$E\left[\int_0^T C_s\, dR_t\right] = E\left[\int_0^T C_s \int_{\mathbb{R}} y\, m_1(X_s, dy, ds)\right]$$

for all nonnegative $\mathcal{F}_t$-adapted processes C_t. Let Radon–Nikodym derivatives be

$$f_i(X_t) = \frac{\int_{\mathbb{R}} y^i m_1(X_t, dy, dt)}{dt},$$

$i = 1, 2$ for a convenience. We also assume that Radon–Nikodym derivatives $f_i(X_t)$ are bounded and $\mathcal{F}_t$-adapted.