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Parameter estimation for a subcritical affine two factor model ☆

Mátyás Barczy [a,*], Leif Döring [b,1], Zenghu Li [c,2], Gyula Pap [d,3]

[a] Faculty of Informatics, University of Debrecen, Pf. 12, H-4010 Debrecen, Hungary
[b] Institut für Mathematik, Universität Zürich, Winterthurerstrasse 190, CH-8057 Zürich, Switzerland
[c] School of Mathematical Sciences, Beijing Normal University, Beijing 100875, People's Republic of China
[d] Bolyai Institute, University of Szeged, Aradi vértanúk tere 1, H-6720 Szeged, Hungary

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ABSTRACT

For an affine two factor model, we study the asymptotic properties of the maximum likelihood and least squares estimators of some appearing parameters in the so-called subcritical (ergodic) case based on continuous time observations. We prove strong consistency and asymptotic normality of the estimators in question.

1. Introduction

We consider the following 2-dimensional affine process (affine two factor model)

$$\begin{cases} \mathrm{d}Y_t = (a - bY_t)\,\mathrm{d}t + \sqrt{Y_t}\,\mathrm{d}L_t, \\ \mathrm{d}X_t = (m - \theta X_t)\,\mathrm{d}t + \sqrt{Y_t}\,\mathrm{d}B_t, \end{cases} \quad t \geq 0, \tag{1.1}$$

where $a > 0$, $b, m, \theta \in \mathbb{R}$, and $(L_t)_{t\geq 0}$ and $(B_t)_{t\geq 0}$ are independent standard Wiener processes. Note that the process $(Y_t)_{t\geq 0}$ given by the first SDE of (1.1) is the so-called Cox–Ingersol–Ross (CIR) process which is a continuous state branching process with branching mechanism $bz + z^2/2$, $z \geq 0$, and with immigration mechanism az, $z \geq 0$. Chen and Joslin (2012) applied (1.1)

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* Corresponding author. Tel.: +36 52 512900; fax: +36 52 512996.
E-mail addresses: barczy.matyas@inf.unideb.hu (M. Barczy), leif.doering@googlemail.com (L. Döring), lizh@bnu.edu.cn (Z. Li), papgy@math.u-szeged.hu (G. Pap).
[1] Tel.: +41 44 63 55892; fax: +41 44 63 55705.
[2] Tel.: +86 10 58802900; fax: +86 10 58808202.
[3] Tel.: +36 62 544033; fax: +36 62 544548.

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for modelling quantitative impact of stochastic recovery on the pricing of defaultable bonds, see their equations (25) and (26).

The process (Y, X) given by (1.1) is a special affine diffusion process. The set of affine processes contains a large class of important Markov processes such as continuous state branching processes and Ornstein–Uhlenbeck processes. Further, a lot of models in financial mathematics are affine such as the Heston model (Heston, 1993), the model of Barndorff-Nielsen and Shephard (2001) or the model due to Carr and Wu (2003). A precise mathematical formulation and a complete characterization of regular affine processes are due to Duffie et al. (2003). These processes are widely used in financial mathematics due to their computational tractability, see Gatheral (2006).

This paper is devoted to estimate the parameters a, b, m and θ from some continuously observed real data set $(Y_t, X_t)_{t \in [0,T]}$, where $T > 0$. To the best knowledge of the authors the parameter estimation problem for multi-dimensional affine processes has not been tackled so far. Since affine processes are frequently used in financial mathematics, the question of parameter estimation for them is of high importance. In Barczy et al. (2013a) we started the discussion with a simple non-trivial two-dimensional affine diffusion process given by (1.1) in the so-called critical case: $b \geq 0$, $\theta = 0$ or $b = 0$, $\theta \geq 0$ (for the definition of criticality, see Section 2). In the special critical case $b = 0$, $\theta = 0$ we described the asymptotic behaviour of least squares estimator (LSE) of (m, θ) from some discretely observed low frequency real data set $X_0, X_1, \ldots, X_n$ as $n \to \infty$. The description of the asymptotic behaviour of the LSE of (m, θ) in the other critical cases $b = 0$, $\theta > 0$ or $b > 0$, $\theta = 0$ remained opened. In this paper we deal with the same model (1.1) but in the so-called subcritical (ergodic) case: $b > 0$, $\theta > 0$, and we consider the maximum likelihood estimator (MLE) of (a, b, m, θ) using some continuously observed real data set $(Y_t, X_t)_{t \in [0,T]}$, where $T > 0$, and the LSE of (m, θ) using some continuously observed real data set $(X_t)_{t \in [0,T]}$, where $T > 0$. For studying the asymptotic behaviour of the MLE and LSE in the subcritical (ergodic) case, one first needs to examine the question of existence of a unique stationary distribution and ergodicity for the model given by (1.1). In a companion paper Barczy et al. (2013b) we solved this problem, see also Theorem 2.5. Further, in a more general setup by replacing the CIR process $(Y_t)_{t \geq 0}$ in the first SDE of (1.1) by a so-called α-root process (stable CIR process) with $\alpha \in (1, 2)$, the existence of a unique stationary distribution for the corresponding model was proved in Barczy et al. (2013b).

In general, parameter estimation for subcritical (also called ergodic) models has a long history, see, e.g., the monographs of Liptser and Shiryaev (2001b, Chapter 17), Kutoyants (2004), Bishwal (2008) and the papers of Klimko and Nelson (1978) and Sørensen (2004). In case of the one-dimensional CIR process Y, the parameter estimation of a and b goes back to Overbeck and Rydén (1997) (conditional LSE), Overbeck (1998) (MLE), and see also Bishwal (2008, Example 7.6) and the very recent papers of Ben Alaya and Kebaier (2012, 2013) (MLE). In Ben Alaya and Kebaier (2012, 2013) one can find a systematic study of the asymptotic behaviour of the quadruplet $(\log(Y_t), Y_t, \int_0^t Y_s \, \mathrm{d}s, \int_0^t 1/Y_s \, \mathrm{d}s)$ as $t \to \infty$. Finally, we note that Li and Ma (2013) started to investigate the asymptotic behaviour of the (weighted) conditional LSE of the drift parameters for a CIR model driven by a stable noise (they call it a stable CIR model) from some discretely observed low frequency real data set. To give another example besides the one-dimensional CIR process, we mention a model that is somewhat related to (1.1) and parameter estimation of the appearing parameters based on continuous time observations has been considered. It is the so-called Ornstein–Uhlenbeck process driven by α-stable Lévy motions, i.e.,

$$\mathrm{d}U_t = (m - \theta U_t)\, \mathrm{d}t + \mathrm{d}Z_t, \quad t \geq 0,$$

where $\theta > 0$, $m \neq 0$, and $(Z_t)_{t \geq 0}$ is an α-stable Lévy motion with $\alpha \in (1, 2)$. For this model Hu and Long investigated the question of parameter estimation, see Hu and Long (2007, 2009a, 2009b).

It would be possible to calculate the discretized version of the estimators presented in this paper using the same procedure as in Ben Alaya and Kebaier (2012, Section 4) valid for discrete time observations of high frequency. However, it is out of the scope of the present paper.

We give a brief overview of the structure of the paper. Section 2 is devoted to a preliminary discussion of the existence and uniqueness of a strong solution of the SDE (1.1), we make a classification of the model (see Definition 2.4), we also recall our results in Barczy et al. (2013b) on the existence of a unique stationary distribution and ergodicity for the affine process given by SDE (1.1), see Theorem 2.5. Further, we recall some limit theorems for continuous local martingales that will be used later on for studying the asymptotic behaviour of the MLE of (a, b, m, θ) and the LSE of (m, θ), respectively. In Sections 3–8 we study the asymptotic behaviour of the MLE of (a, b, m, θ) and LSE of (m, θ) proving that the estimators are strongly consistent and asymptotically normal under appropriate conditions on the parameters. We call the attention that for the MLE of (a, b, m, θ) we require a continuous time observation $(Y_t, X_t)_{t \in [0,T]}$ of the process (Y, X), but for the LSE of (m, θ) we need a continuous time observation $(X_t)_{t \in [0,T]}$ only for the process X. We note that in the critical case we obtained a different limit behaviour for the LSE of (m, θ), see Barczy et al. (2013a, Theorem 3.2).

A common generalization of the model (1.1) and the well-known Heston model (Heston, 1993) is a general affine diffusion two factor model

$$\begin{cases} \mathrm{d}Y_t = (a - bY_t)\, \mathrm{d}t + \sigma_1 \sqrt{Y_t}\, \mathrm{d}L_t, \\ \mathrm{d}X_t = (m - \kappa Y_t - \theta X_t)\, \mathrm{d}t + \sigma_1 \sqrt{Y_t}(\varrho L_t + \sqrt{1 - \varrho^2} \mathrm{d}B_t), \end{cases} \quad t \geq 0, \tag{1.2}$$

where $a, \sigma_1, \sigma_2 > 0$, $b, m, \kappa, \theta \in \mathbb{R}$, $\varrho \in (-1, 1)$, and $(L_t)_{t \geq 0}$ and $(B_t)_{t \geq 0}$ are independent standard Wiener processes. One does not need to estimate the parameters σ_1, σ_2 and ϱ, since these parameters could — in principle, at least — be determined (rather than estimated) using an arbitrarily short continuous time observation of (Y, X), see Remark 2.5 in Barczy and Pap (2013).