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Constructing D-optimal symmetric stated preference discrete choice experiments

Fatih Demirkale [a,*], Diane Donovan [a], Deborah J. Street [b]

[a] School of Mathematics and Physics, Centre for Discrete Mathematics and Computing, University of Queensland, Australia
[b] School of Mathematical Sciences, University of Technology, Sydney, Australia

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ABSTRACT

We give new constructions for DCEs in which all attributes have the same number of levels. These constructions use several combinatorial structures, such as orthogonal arrays, balanced incomplete block designs and Hadamard matrices. If we assume that only the main effects of the attributes are to be used to explain the results and that all attribute level combinations are equally attractive, we show that the constructed DCEs are *D*-optimal.

1. Introduction

Stated choice experiments are widely used to elicit preferences for products and services. A *stated preference discrete choice experiment* (DCE) consists of a set of N choice sets, and each choice set consists of two or more options. Each respondent is shown each choice set in turn and asked to choose one of the options presented in the choice set. The number of options in a choice set, m, is called the *choice set size*. A thorough discussion of DCEs may be found in Street and Burgess (2007).

We start by describing a typical example. In this study energy efficient lightbulbs are described by five attributes, each of which has three levels, and the main effects of these attributes are of interest. The attributes are *Quality of light* with three levels (cool white, white and warm white), *Lifetime of bulb* with three levels (6000, 9000 and 12 000 h), *Recycling available* with three levels (at a shopping mall, kerbside and at a recycling depot), *Time to reach full brightness* with three levels (5 s, 10 s and 15 s) and *Replacement cost* with three levels ($3.50, $7 and $10.50).

Each choice set consists of three options and an example choice set appears in Table 1.

In this paper, as is often the case in practice, we assume that the options under consideration are each described by k attributes. We further assume that each attribute takes one of ℓ levels and that these levels are represented by the elements of $\mathbb{Z}_\ell$. Thus there are $L=\ell^k$ possible options in total and we let F be the set of all possible options. Each choice set contains m distinct options. We will denote a choice set by T_i, $1 \le i \le N$, with options $T_{i,1}, T_{i,2}, \ldots, T_{i,m}$. Given that the options

* Corresponding author. Tel.: +61 733656137.
E-mail address: fatih.demirkale@uqconnect.edu.au (F. Demirkale).

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Table 1
A sample choice set.

Attribute	**Option 1**	**Option 2**	**Option 3**
Quality of light	Warm white	Cool white	White
Lifetime of bulb (h)	6000	9000	12,000
Recycling	Shopping mall	Kerbside	Recycling depot
Time to full brightness (s)	5	10	10
Cost	$7	$3.50	$7
Your choice	$	$	$

are described by the levels of each of k attributes, each $T_{i,j}$ is a k-tuple and we let $T_{i,j}=(T_{i,j}^{(1)},T_{i,j}^{(2)},\ldots,T_{i,j}^{(k)})$, where $T_{i,j}^{(q)}\in\mathbb{Z}_\ell$ for $1\le q\le k$. With this notation in place, we define a DCE as follows:

Definition 1.1. The array below corresponds to the DCE E. Each row corresponds to one of the N choice sets, and each column to one of the m options, where each choice set is in some fixed, but arbitrary, order. The entries in the array are k-tuples with elements from $\mathbb{Z}_\ell$

$$E=\begin{bmatrix} T_{1,1} & T_{1,2} & \cdots & T_{1,m} \\ T_{2,1} & T_{2,2} & \cdots & T_{2,m} \\ & & \vdots & \\ T_{N,1} & T_{N,2} & \cdots & T_{N,m} \end{bmatrix}. \qquad \square$$

The class of all such DCEs will be denoted by $\mathcal{E}$. Designs in $\mathcal{E}$ may contain repeated choice sets.

For the remainder of this paper all DCEs will be assumed to be of the form given in Definition 1.1.

We will assume that the options in F, the set of all possible options, are ordered lexicographically and are labeled by $0,1,\ldots,L-1$. Thus the first option is $(0,0,\ldots,0)$, the second option is $(0,0,\ldots,1)$ and the final option is $(\ell-1,\ell-1,\ldots,\ell-1)$. If $0\le u\le \ell^k-1=L-1$ and $u=u^{(1)}\ell^{k-1}+u^{(2)}\ell^{k-2}+\cdots+u^{(k)}$, we write $u=[u^{(1)},u^{(2)},\ldots,u^{(k)}]$, establishing a one-to-one correspondence between the options and their labels.

Not all groupings of options into choice sets, and groupings of choice sets into DCEs, are equally good. Many ways to compare DCEs have been proposed and most are based on functions of the variance-covariance matrix of the parameter estimates, most frequently on the determinant of that matrix and we will use that function in this paper.

When making choices subjects must employ a decision rule. As in Burgess and Street (2005) (or Street and Burgess, 2007, Chapter 3) we will assume that a multinomial logit (MNL) model is the appropriate discrete choice model to model how subjects are making choices. Hence we know that under the assumption of equal choice probabilities (equivalently, a parameter vector of 0s), the information matrix of the options $\Lambda=[\Lambda(r,s)]$, of order L and with rows and columns indexed by $0,1,\ldots,L-1$, is given by

$$\Lambda(r,s)=\begin{cases} \dfrac{m-1}{m^2N}n_r & \text{if } r=s, \\ -\dfrac{1}{m^2N}n_{r,s} & \text{if } r\neq s, \end{cases}$$

where n_r is the number of times that option r appears in the DCE and $n_{r,s}$ is the number of times that options r and s are in the same choice set in the DCE.

If only the main effects of the attributes are of interest, then only we need the information matrix corresponding to those effects. Thus we find a contrast matrix for these effects, say, B, and evaluate $C=B\Lambda B'$ which is the information matrix for the main effects only. We assume that the rows of B represent a set of orthogonal contrasts.

As in Burgess and Street (2005), we let B_ℓ be a normalized contrast matrix for main effects for an attribute with ℓ levels. This means that B_ℓ is a matrix of order $(\ell-1)\times\ell$ with rows that sum to 0 and which satisfies $B_\ell B_\ell{}'=I_{\ell-1}$. (Note the B_ℓ is not unique for $\ell\ge 3$ but that the determinant of C is independent of the particular set of contrasts used in B_ℓ.) We index the rows of B_ℓ using $1,\ldots,\ell-1$ and we index the columns of B_ℓ using the elements of $\mathbb{Z}_\ell$. Let $\mathbf{j}_\ell$ be a $1\times\ell$ row vector of the ones and let $\otimes$ denote the Kronecker product. Then a normalized contrast matrix for the main effects for a DCE with k attributes each with ℓ levels is

$$B=\begin{bmatrix} B_\ell\otimes\frac{1}{\sqrt{\ell}}\mathbf{j}_\ell\otimes\cdots\otimes\frac{1}{\sqrt{\ell}}\mathbf{j}_\ell \\ \frac{1}{\sqrt{\ell}}\mathbf{j}_\ell\otimes B_\ell\otimes\cdots\otimes\frac{1}{\sqrt{\ell}}\mathbf{j}_\ell \\ \vdots \\ \frac{1}{\sqrt{\ell}}\mathbf{j}_\ell\otimes\frac{1}{\sqrt{\ell}}\mathbf{j}_\ell\otimes\cdots\otimes B_\ell \end{bmatrix}. \tag{1}$$

We see that B is of order $k(\ell-1)\times L$.