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Improving the finite sample performance of tests for a shift in mean

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ABSTRACT

It is widely known that structural break tests based on the long-run variance estimator, which is estimated under the alternative, suffer from serious size distortion when the errors are serially correlated. In this paper, we propose bias-corrected tests for a shift in mean by correcting the bias of the long-run variance estimator up to $O(1/T)$. Simulation results show that the proposed tests have good size and high power.

1. Introduction

Testing for structural breaks has been a longstanding problem and various tests have been proposed in the econometric and statistical literature. One of the frequently used tests for parameter constancy against the general alternative is the CUSUM test based on recursive residuals proposed by Brown et al. (1975), and this test was further developed based on OLS residuals by Ploberger and Krämer (1992). By specifying a random walk as the alternative, optimal tests for parameter constancy were investigated by Nyblom and Mäkeläinen (1983), Nyblom (1986, 1989), and Nabeya and Tanaka (1988), among others, while the point optimal test for general regression models was studied by Elliott and Müller (2006). On the other hand, it is often the case that a one-time structural change with an unknown change point is considered as the alternative and the sup-type test by Andrews (1993) and the mean- and exponential-type tests developed by Andrews and Ploberger (1994) and Andrews et al. (1996) are widely used in practical analyses. For a general discussion on structural changes, see, for example, Csörgő and Horváth (1997), Perron (2006), and Aue and Horváth (2013).

In practice, when we test for structural breaks in time-series models, we need to take serial correlation into account, and thus we have to estimate the long-run variance of the errors. If we estimate the long-run variance under the null hypothesis of no structural breaks, then it is known that the above tests suffer from the so-called non-monotonic power problem, that is, the power initially rises under the alternative, but as the magnitude of the break increases, the power eventually falls and tends to zero. This problem was investigated by Vogelsang (1999), Crainiceanu and Vogelsang (2007), Deng and Perron (2008), and Perron and Yamamoto (forthcoming). The reason for this problem is that the long-run variance estimator takes significantly large values as the magnitude of the break increases.

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On the other hand, if we estimate the long-run variance under the alternative, then the tests suffer from size distortion; they tend to over-reject the null hypothesis. This is because the long-run variance is under-estimated, so that the test statistics tend to take large values under the null hypothesis of no break.

In order to cope with the problem associated with the estimation of the long-run variance, several methods have been proposed. Kejriwal (2009) proposed to estimate the long-run variance using the residuals under both the null and alternative hypotheses. By using this hybrid estimator, we can reduce size distortion, but the power becomes extremely low when the error is strongly serially correlated. Juhl and Xiao (2009) proposed to estimate the long-run variance using the residuals of the nonparametric regression to mitigate the non-monotonic power problem. However, the finite sample performance of this test crucially depends on the choice of the bandwidth in the nonparametric regression. While these papers tried to improve the accuracy of the long-run variance estimator, there are several methods with which we do not have to consistently estimate the long-run variance. Sayginsoy and Vogelsang (2011) and Yang and Vogelsang (2011) proposed fixed-b sup-Wald and fixed-b sup-LM tests, respectively, which are robust to I(0)/I(1) errors. The fixed-b framework is based on Kiefer and Vogelsang (2005), which used an inconsistent long-run variance estimator where the bandwidth is proportional to the sample size. The fixed-b sup-Wald and sup-LM tests have relatively good sizes under the null hypothesis, but there is a loss of power due to the inconsistent estimation of the long-run variance. On the other hand, Shao and Zhang (2010) proposed a self-normalized test based on the CUSUM test. The basic idea of self-normalization is similar to the fixed-b approach. Although the finite sample performance of these tests are improved, compared to the frequently used tests, such as the original CUSUM and sup-type tests, the existing methods do not seem to be satisfactory in terms of both size and power.

In this paper, we develop an accurate long-run variance estimator and propose to use it to improve the finite sample property of the structural change tests. This estimator can be obtained by correcting the bias up to $O(T^{-1})$, where T is the sample size. The key feature of our method is that bias correction is achieved by taking a structural break into account. The advantage of our method is that tests with our long-run variance estimator can control the empirical size well, while maintaining high power. The simulation results show that the proposed tests have a higher power than other tests, such as the fixed-b test. Moreover, the power difference between our bias-corrected tests and the original (bias-uncorrected) tests is very minor, and it becomes negligible as the sample size increases. This result is in contrast to some other tests, which suffer from asymptotic power loss.

The remainder of this paper is organized as follows. In Section 2, we introduce the model and the test statistic. The derivation of the bias term is discussed in Section 3, and the bias correction method is explained in Section 4. The case with general error processes is discussed in Section 5. Simulation results are given in Section 6, and Section 7 concludes the paper. All mathematical proofs are delegated to the Appendix.

2. Model and test statistic

Let us consider the following mean-shift model:

$$y_t = \mu + \delta \cdot DU_t(T_b^0) + u_t, \quad t = 1, \ldots, T, \tag{1}$$

where $DU_t(T_b^0) = 1\{t > T_b^0\}$, and $1\{\cdot\}$ is the indicator function. We assume that u_t is a zero-mean stationary process and that the break date T_b^0 is unknown.

The testing problem is

$$H_0: \quad \delta = 0 \quad \text{vs.} \quad H_1: \quad \delta \neq 0. \tag{2}$$

Under H_0, there is no shift in mean, whereas under H_1, there is a one-time break.

In order to test for a shift in mean, we need to estimate the long-run variance of u_t defined by $\omega = \sum_{\ell=-\infty}^{\infty} E(u_t u_{t-\ell})$ for the scale adjustment, which can be consistently estimated by the kernel method. As it is known that tests with ω estimated under the null hypothesis suffer from the non-monotonic power problem, as pointed out by Vogelsang (1999), we exclude the case where the long-run variance is estimated under the null hypothesis, and focus on the case where it is estimated under the alternative of a one-time break. That is, we consider the following kernel estimator of ω as a benchmark:

$$\hat{\omega}(T_b) = \hat{\gamma}_0 + 2\sum_{j=1}^{T-1} k\left(\frac{j}{m}\right)\hat{\gamma}_j, \tag{3}$$

where $k(\cdot)$ is the kernel function, m is the bandwidth, $\hat{\gamma}_j$ is the estimator of the jth autocovariance of u_t defined by $\hat{\gamma}_j = T^{-1}\sum_{t=j+1}^{T} \hat{u}_t \hat{u}_{t-j}$, the residuals $\hat{u}_t$ are obtained under the alternative with the supposed break date T_b, and

$$\hat{u}_t = \begin{cases} y_t - \bar{y}_1 & \text{for } t = 1, \ldots, T_b, \\ y_t - \bar{y}_2 & \text{for } t = T_b + 1, \ldots, T, \end{cases} \tag{4}$$

where $\bar{y}_1 = T_b^{-1}\sum_{t=1}^{T_b} y_t$ and $\bar{y}_2 = (T - T_b)^{-1}\sum_{t=T_b+1}^{T} y_t$. Note that T_b is specified by a researcher and it is not necessarily consistent with T_b^0. We suppress the dependency of $\hat{\gamma}_j$ and $\hat{u}_t$ on T_b for notational simplicity.