Journal of Statistical Planning and Inference 143 (2013) 1512–1526

Contents lists available at SciVerse ScienceDirect

Journal of Statistical Planning and Inference

journal homepage: www.elsevier.com/locate/jspi

On control charts for monitoring the variance of a time series

Taras Lazariv *, Wolfgang Schmid, Svitlana Zabolotska

Department of Statistics, European University Viadrina, PO Box 1786, D-15207 Frankfurt (Oder), Germany

ARTICLE INFO

Article history:
Received 14 September 2012
Received in revised form
18 February 2013
Accepted 8 April 2013
Available online 18 April 2013

Keywords:
Control charts
CUSUM charts
Generalized likelihood ratio
SPRT
Shiryaev–Roberts procedure
Variance changes
Time series
Statistical process control
Sequential detection

ABSTRACT

In this paper we derive control charts for the variance of a Gaussian process using the likelihood ratio approach, the generalized likelihood ratio approach, the sequential probability ratio method and a generalized sequential probability ratio procedure, the Shiryaev–Roberts procedure and a generalized modified Shiryaev–Roberts approach. Recursive presentations for the calculation of the control statistics are given for autoregressive processes of order 1. In an extensive simulation study these schemes are compared with existing control charts for the variance. In order to asses the performance of the schemes both the average run length and the average delay are used.

1. Introduction

In many applications we are faced with the problem to detect changes over time in an observed process. Because usually a change should be detected fast sequential methods are more appropriate in such a situation. The most important tools for monitoring a process are control charts (cf. Stoumbos et al., 2000). Control charts are successfully applied in engineering for a long time (e.g., Lawson and Kleinman, 2005; Frisén, 2007). In the last 20 years many further applications in different areas have been studied like, e.g., in public health, economics, environmental sciences. In that context the underlying processes have a more complicate structure and are mostly modeled by a time series. Alwan and Roberts (1988) showed that control charts for independent variables cannot be directly applied to time series. They proposed to use residual charts, i.e. to transform the original observations such that the transformed observations are independent and to apply the well-known control charts to these residuals. Residual charts have been studied by several authors (e.g., Harris and Ross, 1991; Wardell et al., 1994; Lu and Reynolds, 1999). Another possibility is to directly monitor the observed process. The behavior of the Shewhart chart for time series was studied in Schmid (1995). An extension of the exponentially weighted moving average (EWMA) chart of Roberts (1959) to time series was proposed by Schmid (1997a). Cumulative sum (CUSUM) charts for time-dependent processes have been studied among others by Nikiforov (1975), Schmid (1997b) and Knoth and Frisén (2012). An overview on control charts for time series is given in Knoth and Schmid (2004) and Okhrin and Schmid (2007).

* Corresponding author. Tel.: +49 17673298918.
E-mail addresses: lazariv.taras@gmail.com, lazariv@europa-uni.de (T. Lazariv).

0378-3758/$ - see front matter © 2013 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.jspi.2013.04.001

Most of the literature on that topic is dealing with the monitoring of the mean behavior of the observed process. Here we want to focus on the surveillance of the variance of a time series. We are interested to detect an increase in the variance. For instance, such a question is of great importance in economics where the variance is the most applied measure for the risk and an early detection of a change in the risk behavior of an asset is an important information for an analyst. The first EWMA control chart for time series was introduced by MacGregor and Harris (1993). Schipper and Schmid (2001) introduced several one-sided variance charts for stationary processes, however, their main focus was in the area of nonlinear time series. In many applications the control statistic for independent processes is used and the independent process is replaced by the time series. Thus the structure of the time series is not taken into account for the derivation of the control statistic. This is a disadvantage of this procedure.

In this paper we derive control charts for the variance of a Gaussian process by making use of the likelihood ratio approach (LR), the sequential probability ratio test of Wald (SPRT), and the Shiryaev–Roberts (SR) procedure. For deriving these charts it is first assumed that the size of the change is known. Thus all obtained charts depend on a reference value which has to be suitably chosen in advance. This is sometimes a drawback in applications. We consider generalized control charts as well. They are obtained via the generalized LR, SPRT, and modified SR procedure. The great advantage of these schemes is that they do not depend on a reference value. It has to be emphasized that our results are quite general and cover all autoregressive moving average processes with a Gaussian white noise.

In Section 2 the underlying model of the paper is introduced. It is explained how in our paper the target process and the observed process are related with each other. In Section 3 the CUSUM control chart for the variance in the independent case is briefly presented and a new CUSUM control scheme for Gaussian processes is derived over the LR approach. In an example we consider the special case of autoregressive processes of order 1 and it is shown that in that case the control statistics can be calculated recursively. In Section 4 a CUSUM variance chart is derived by the SPRT and the result is a residual chart. In Section 5 the Shiryaev–Roberts method is used to get a control chart for the variance. In Sections 6–8 generalized control schemes are derived. In Section 6 we use the generalized LR method, in Section 7 a generalization of the SPRT approach, and in Section 8 a generalization of a modified version of the SR method is obtained.

In an extensive simulation study these control schemes are compared with each other assuming that the underlying target process is an autoregressive process of order 1 (AR(1)) (Section 9). As a measure for the performance of a control scheme the average run length (ARL) and the average delay are taken. All charts are calibrated such that the in-control ARL is the same if no change is present. Our results show that except the SR chart all other schemes with a reference value have the smallest out-of-control ARL if the reference value is equal to the true value of the change. It turns out that the generalized modified SR chart has the smallest ARL if the change is small. It is even better than the charts with the optimal reference value. For medium and larger changes the LR chart and the SPRT chart provide better results provided that the reference value is not dramatically smaller than the true change. Except the generalized SPRT scheme for all charts the worst average delay is equal to the average run length. The limit of the average delay seems to be the smallest for the GMSR chart. This scheme must be preferred for larger changes if the change arises at a later time point.

2. Modeling

The aim of statistical process control (SPC) is to detect structural deviations within a process over a time. It is examined whether the present observations can be considered as realizations of a given target process $\{Y_t\}$. The procedure is a sequential one. The observations (samples) are analyzed consecutively. It is desirable to detect a change as quickly as possible after its occurrence. Of course there are various types of changes which may influence the target process. In this paper we focus on the detection of an increase in the variance. Such a problem arises in practice very often. For instance, the variance is considered as a risk measure in economics and thus an increasing variance is a hint that the risk of an asset is getting larger. In engineering the variance reflects the quality of a production process and an increase is a bad sign since the production is getting worse.

Let $\{Y_t\}$ be a (weakly) stationary process with mean μ and autocovariance function $Cov(Y_t, Y_{t+h}) = \gamma(h)$. In what follows it is assumed that the relationship between the target process $\{Y_t\}$ and the observed process $\{X_t\}$ is given by

$$X_t = \begin{cases} Y_t & \text{for } 1 \le t < \tau \\ \mu + \Delta(Y_t - \mu) & \text{for } t \ge \tau \end{cases}, \tag{1}$$

for $t \in \mathbb{Z}$ with $\Delta > 1$ and $\tau \in \mathbb{N} \cup \{\infty\}$. Thus a change in the scale appears at position τ if $\tau < \infty$. The observed process is said to be out of control. Else, if $\tau = \infty$, then $\{X_t\}$ is called to be in control. Here it is assumed that at a given time point exactly one observation is available.

Note that the change in the scale does not influence the mean structure. It holds that $E(X_t) = \mu$ as well in the in-control state as in the out-of-control state. Moreover, we get that

$$Var(X_t) = \begin{cases} Var(Y_t) & \text{for } 1 \le t < \tau \\ \Delta^2 \, Var(Y_t) & \text{for } t \ge \tau \end{cases},$$