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Estimation and inference for exponential smooth transition nonlinear volatility models

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ABSTRACT

A family of threshold nonlinear generalised autoregressive conditionally heteroscedastic models is considered, that allows smooth transitions between regimes, capturing size asymmetry via an exponential smooth transition function. A Bayesian approach is taken and an efficient adaptive sampling scheme is employed for inference, including a novel extension to a recently proposed prior for the smoothing parameter that solves a likelihood identification problem. A simulation study illustrates that the sampling scheme performs well, with the chosen prior kept close to uninformative, while successfully ensuring identification of model parameters and accurate inference for the smoothing parameter. An empirical study confirms the potential suitability of the model, highlighting the presence of both mean and volatility (size) asymmetry; while the model is favoured over modern, popular model competitors, including those with sign asymmetry, via the deviance information criterion.

1. Introduction

In financial modelling and risk management, the dynamic volatility of financial return data is of prime importance. Engle (1982) and Bollerslev (1986), respectfully, developed the Autoregressive Conditional Heteroskedasticity (ARCH) model, and the generalised ARCH (GARCH), to capture such dynamic volatility and the well-known volatility clustering trait. Over the last two decades the GARCH model has been extended, modified and refined to handle various other aspects of financial returns, such as excess kurtosis, volatility asymmetry and mean asymmetry. This extended family has become the most popular for financial return and volatility modelling. We consider a modification to a recent addition to this family in this paper.

To capture nonlinear dynamics, Tong (1978) proposed the threshold autoregressive (TAR) model. The concept was applied to the GARCH family by Glosten et al.'s (1993) GJR-ARCH model, where a single ARCH parameter changed in response to negative shocks; Zakoian (1994) extended to the full T-GARCH model. Such models capture volatility asymmetry, via the leverage effect hypothesis of Black (1976). Mean asymmetry was also included in Li and Li's (1996) double threshold DT-ARCH process. The model became fully double threshold GARCH (DT-GARCH) via Brooks (2001); extended by Chen et al. (2003) to include exogenous threshold variables and nonlinear mean spill-over effects. Studies such as these, as well as Chen et al. (2005, 2006), show that double threshold nonlinear models are useful and favoured, over their simpler counterparts, for many financial return series, using model selection and forecast accuracy criteria.

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The nonlinear models discussed have step function regime transitions. Such transitions can be generalised to be smooth, as in Bacon and Watts (1971), who proposed a smooth transition (ST) nonlinear regression model. Such transitions make sense from a financial return viewpoint; e.g. Black's (1976) idea that volatility is higher when asset values are falling, may not be well captured by a sharp transition, since when asset value falls by 0.01% the volatility may not subsequently rise at all, while volatility may significantly rise following a 2% fall in value, and may rise even moreso following a 10% drop in value. Such behaviour can be captured by a smooth transition function. Chan and Tong (1986) proposed the ST autoregressive (STAR) model for homoscedastic data; further developed by Luukkonen et al. (1988), Granger and Teräsvirta (1993) and Teräsvirta (1994). In practice, the ST function is the logistic, the exponential or an alternate cumulative distribution function; i.e.

$$\text{Logistic ST (LST) function :} F(z;\gamma,c) = \{1+\exp[-\gamma(z-c)]\}^{-1}, \quad \gamma>0,$$

$$\text{Exponential ST (EST) function :} F(z;\gamma,c) = 1-\exp[-g(\gamma(z-c))], \quad \gamma>0, \tag{1}$$

where z is the transition variable, c is the threshold limit and $g(\cdot)$ is a non-negative valued function.

The logistic ST (LST) is an odd function, and is used to capture sign asymmetry: asymmetric responses to positive and negative values of $z-c$. Teräsvirta and Anderson (1992) and Teräsvirta (1994) applied the STAR model with an LST to financial data, finding evidence of sign asymmetry in the mean. However, the exponential ST (EST) is an even function; this captures size asymmetry, or asymmetric responses to the magnitude of $z-c$. See Granger and Teräsvirta (1993) and Teräsvirta (1994) for applications of the STAR with EST model. Michael et al. (1997), Sarantis (1999), and Taylor et al. (2001) applied the STAR with EST model to exchange rates, finding evidence of size asymmetry in mean.

Anderson et al. (1999) developed an asymmetric nonlinear smooth transition (ANST-)GARCH model, to capture self-exciting sign asymmetric volatility processes. The volatility equation was

$$h_t = (1-F(a_{t-1}))h_t^{(1)} + F(a_{t-1})h_t^{(2)},$$

$$h_t^{(\ell)} = \alpha_0^{(\ell)} + \alpha_1^{(\ell)}a_{t-1}^2 + \beta_1^{(\ell)}h_{t-1}, \quad \ell = 1,2,$$

where $F(\cdot)$ is the logistic ST function (1), with $z=a$ (the shock or residual) and threshold limit set as $c=0$. Nam et al. (2001) extended this model to also capture self-exciting mean asymmetry, adding a STAR mean equation, again setting $c=0$. Gerlach and Chen (2008) proposed the general double ST-GARCH (DST-GARCH) model, with LST function, adding exogenous nonlinear mean effects, considering endogenous and exogenous threshold variables and allowing fat-tailed (Student-t) errors. These papers all considered sign asymmetry only. However, Sarantis (1999), Leeves (2007) and Chen et al. (2008) illustrated the importance of size asymmetry when modelling and forecasting financial return volatility. In this paper, we thus consider the DEST-GARCH model: a double ST-GARCH with an exponential ST function to capture size asymmetry.

Not much work has been done on estimation of either the speed of transition parameter γ, the threshold limit c or on the delay lag parameter d, i.e. $z=z_{t-d}$ in (1); however, each can be problematic, especially under a frequentist and/or likelihood only approach. This is because: (i) the likelihood for γ is non-integrable, since it is well-defined for $\gamma=\infty$; while both $\gamma=0$ and ∞ present an identification problem, as discussed in Section 3; (ii) the likelihood is non-differentiable, and often multimodal (Giordani et al., 2007), in terms of c, leading Bauwens et al. (1999) to remark that "any classical measure of uncertainty for c seems unfeasible"; and (iii) d is a discrete variable, making numerical likelihood optimisation an issue. Frequentist approaches for estimating threshold models usually involve setting c,d (e.g. $c=0$, $d=1$), or choosing c,d via some information criterion (e.g. Li and Li, 1996), then estimating parameters conditional upon these choices. Further, both frequentist and Bayesian estimates of γ are often large, so that a step transition is implied (e.g. Nam et al., 2001; Lopes and Salazar, 2006), and regardless, many reported estimates have very large standard errors (e.g. Sarantis, 1999; Chelley-Steeley, 2005; Lubrano, 2001), so that the question of whether $\gamma=0$ is pertinent, but remains unanswered. These problems are quite manageable under a Bayesian approach, taken in this paper, but require some set-up work, as discussed in Section 3.

Lubrano (2001) discussed Bayesian inference in an ST-GARCH model, noting the problem in (i), suggesting some informative, proper prior distributions for γ, proving that the resulting posteriors were proper and integrable. Gerlach and Chen (2008) further suggested a log-normal prior, applicable for the LST function, and designed an efficient, adaptive Markov chain Monte Carlo (MCMC) sampling scheme for a DST-GARCH model (with LST function) that solved the problem of identifiability as $\gamma \to 0$. We build on that work, extending and adapting the prior to cover an EST function, while also adapting the sampling scheme, for two types of EST function. Further, while Gerlach and Chen (2008) considered the Student-t likelihood directly, here the scale mixture of normal representation of the Student-t is exploited, to preserve a conditionally Gaussian likelihood function and perhaps obtain increases in efficiency.

Section 2 reviews the DEST-GARCH model and the EST function to motivate potential prior choices; Section 3 presents the prior specifications and the MCMC sampling scheme, while Section 4 illustrates the methods via a simulation study. Section 5 presents an empirical study of the Japanese market. Section 6 concludes.