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Ruin probabilities for Bayesian exchangeable claims processes

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ABSTRACT

Among the driving assumptions in classical collective risk models, the independence among claims is frequently violated by real applications. Therefore, there is an evident need of models that relax such a restriction. We undertake the exchangeable claims platform and obtain some results for the infinite time ruin probability. The main result is that the ruin probability under the exchangeable claims model can be represented as the expected value of the ruin probabilities corresponding to certain independent claims cases. This allows us to extend some classical results to this dependent claims scenario. The main tool is based on the de Finetti's representation theorem for exchangeable random variables, and as a consequence a natural Bayesian modeling feature for risk processes becomes available. In particular, an interesting redefinition of the net profit condition is necessary.

1. Introduction

The cornerstone risk model in collective insurance contracts is the Cramér–Lundberg (CL) model (Lundberg, 1909, 1919; Cramér, 1969, 1930). That is, where the evolution of the cash flow of a company, or of a specific business, over time, is modeled by the *risk reserve process* $\{R_t^u\}_{t\geq 0}$, with

$$R_t^u = u + pt - \sum_{i=1}^{N_t} Y_i. \tag{1}$$

Here, the positive value u denotes the *initial surplus*, p is the constant *premium rate*, $N = \{N_t\}_{t\geq 0}$ is a homogeneous Poisson process of rate λ, and $Y = \{Y_i\}_{i\geq 0}$ is the *claims sizes process*, which is assumed to be independent of N. The variable N_t is interpreted as the number of claims received by the company during the time interval $(0, t]$ and the random variable Y_i represents the magnitude of the ith claim, thus typically assumed to follow a distribution supported on $\mathbb{R}_+$. The main question regarding the above risk reserve process, is the computation of the ruin probability for a given initial surplus value. That is, provided that a profitable business is undertaken, i.e. $\lim_{t\uparrow\infty} R_t^u = \infty$, the aim is to find

$$\psi(u) = \mathbb{P}\left[\inf_{t\geq 0} R_t^u < 0\right], \tag{2}$$

namely the probability that the reserve reaches negative values in a finite time.

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The literature on the topic is vast with some excellent expositions found in references such as Beard et al. (1969), Dickson et al. (1992), Rolski et al. (1999) and Asmussen and Albrecher (2010), among many others. Furthermore, several generalizations mainly featuring more realistic considerations for the counting processes N and more general premium functions, can be easily found in the literature, see for instance Grandell (1997). However, for most of these, as for the classical CL model, independence among claims is a persistent assumption.

The wide variety of risk products and their complexity, together with the current international needs such as the Solvency II directive (see Linder and Ronkainen, 2004), require models able to relax the strong requirement of independence among claims. It is easy to envisage insurance contracts where positively dependent claims are present, and thus inducing an increment of the aggregated claims. For example, using data from Friedman and Companies (1987), Müller and Pflug (2001) observe that claims caused by tornadoes exhibit a significative positive correlation. Indeed, violations of the independence assumption have been noted by some authors, e.g. Ambagaspitiya (2009), Biard et al. (2008) and Abbas et al. (2012). This, either puts in risk the solvency of a particular company or locates it out of business, provided a better offer by other company is on the market. Furthermore, insurance regulations typically set a reserve threshold where companies are allowed to operate, imposing competing rules in the market. The reserve quantification is intrinsically linked with the ability of the company to avoid bankruptcy, which in turn is subject to a robust evaluation of the ruin probability (2).

Up to date literature offers only a few risk models that incorporate dependence among claims. For example: Albrecher and Boxma (2004) allow the time between claims to depend on the previous claim magnitude, relaxing with this the independence assumption; Gerber (1982) and Promislow (1991) assume a linear ARMA model for the claim sizes process. Mikosch and Samorodnitsky (2000a) use a stationary stable process for the same purpose and Cossette and Marceau (2000) model dependence through a Poisson shock process. Although, in most of these generalizations the corresponding implications on the ruin probability were explored, the complexity inherent to such dependence structures complicates the obtention of explicit formulae. In most cases, only some bounds or asymptotic results for the ruin probability are provided. For instance, in the case of light-tailed claim sizes, Müller and Pflug (2001) present a Lundberg-type limiting result by assuming conditions on the probability-generating function, Nyrhinen (1998, 1999) obtain rough exponential estimates for ruin probabilities using large deviations techniques, and Albrecher and Kantor (2002) study the behavior of the Lundberg exponent as a function of a dependence measure using three simulation methods of the ruin probability. For heavy-tailed claims Asmussen et al. (1999) extend a classical result on the distribution's tail of the maximum of a random walk, and Mikosch and Samorodnitsky (2000a,b) evaluate the asymptotic behavior of the ruin probability for some ergodic stable processes. See also Asmussen and Albrecher (2010, Ch. XIII) and the references therein.

In addition, it is worth mentioning the Markov-modulated reserve models considered in Lu and Li (2005), Asmussen and Rolski (1994), Reinhard (1984) and Asmussen and Albrecher (2010, Ch. VII). Roughly speaking, in these models the claims distribution, frequency of claims and premium rate, are driven by a Markov process with m states. Unfortunately, only for the case $m = 2$ the ruin probability has a tractable expression.

Our purpose is to further elaborate in this direction by assuming claims in the CL setting are exchangeable. With a different purpose in mind, namely the estimation of the process, this idea was first explored by Mena and Nieto-Barajas (2010). However, apart of some finite-horizon simulation scenarios, little was said about the probability of ruin. Hence, here we look for various analytical results equivalent to those available for the ruin probability in the CL model. In particular, we observe that, due to the identically distributed property featuring exchangeable sequences, some of the marginal properties carry over to the exchangeable case. Most importantly, explicit equivalences of various results, e.g. a generalization of the Pollaczek–Khinchine formula and a generalized Lundberg inequality are also presented. The central ingredient in our proposal is the Bayesian construction of exchangeable sequences. This, apart of leading to some explicit results, brings out an appealing modeling component to the CL approach.

The layout of the paper is as follows: In Section 2 we present various results for the general exchangeable claims case. Section 3 we extend the idea of net profit condition to our setting. These results are then applied to some specific claim scenarios in Section 4. Some final concluding remarks are deferred to Section 5.

2. The Cramér–Lundberg model with exchangeable claims

When going away from the independence assumption, perhaps the most natural property to look at is exchangeability. Let us recall that a sequence of random variables, is said to be exchangeable if $\{Y_1, \ldots, Y_n\} \overset{\mathrm{d}}{=} \{Y_{\pi_1}, \ldots, Y_{\pi_n}\}$ for every $n \geq 1$ and any permutation, π, of $\{1, \ldots, n\}$. Indeed, following the de Finetti's representation theorem, an infinite sequence of $\mathbb{Y}$-valued random variables, $Y = \{Y_i\}_{i\geq 0}$, is said to be exchangeable if and only if it exists a distribution μ on the space $\mathscr{P}$ of all probability measures on $\mathbb{Y}$ such that

$$\mathbb{P}[Y_1 \in A_1, \ldots, Y_n \in A_n] = \int_{\mathscr{P}} \prod_{i=1}^{n} Q(A_i)\mu(dQ), \tag{3}$$

for any $n \geq 1$ and any Borel sets A_i, with $i = 1, \ldots, n$. Clearly, the measure μ, sometimes referred to as the de Finetti's measure of Y, controls the dependence of the conditionally iid sequence. In Bayesian terms, this means that we need to propose a (prior) distribution, μ, on the uncertain, Q, in order to have a specific dependence structure for Y.