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Discussion

Discussion of "Three-phase optimal design of sensitivity experiments" by Wu and Tian

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Almost all of the numerical methods used for finding the root of a function require either a starting point or a range that contains the root. However, there is no easy way to find a good starting point and/or the range. This problem is exacerbated when the function can be observed only with error. The problem becomes even worse in the extreme case of binary data. This is a very challenging problem! Professors Wu and Tian are to be congratulated for attempting to solve this problem and coming up with a reasonable solution. This forms the first phase of their three-phase procedure and the authors rightly point out it as the most novel step in their procedure.

Many different things can be done in the remaining two phases. Specifically, the authors use a D-optimal criterion in the second phase to spread out the points for the efficient estimation of the model parameters and use the Robbins–Monro–Joseph (RMJ) procedure for the third phase to zero-in to the desired quantile (the root of the distribution function). Although their specific choices may not be optimal, the framework they have proposed is quite innovative and opens the door for more research.

In this discussion, we will investigate on an alternative stochastic approximation method for the third phase.

1. An alternative approach for phase III

The Robbins–Monro (1951) method is mainly designed for continuous responses and therefore, Joseph (2004) has modified it to suit binary data. Alternatives to this choice exist, such as Wu′s logit-MLE method (Wu, 1985). We are curious to see how this method performs compared to RMJ. To avoid tedious programming of the first two phases, we have decided to implement it from the beginning of the sequential procedure (i.e., starting with $n=1$). However, MLE of the parameters does not exist until an overlapping pattern is observed and therefore, such an implementation is not feasible. One approach to overcome this existence problem is to postulate informative priors on the parameters (Joseph et al., 2007; Dror and Steinberg, 2008).

Let

$$F(x|\gamma)=\frac{e^{(x-\mu)/\sigma}}{1+e^{(x-\mu)/\sigma}},$$

where $\gamma=(\mu,\sigma)$ and assume the independent priors

$$\mu\sim N(\mu_0,\tau^2) \quad \text{and} \quad \sigma\sim Exponential(\theta_0).$$

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Suppose we have observed, (x_i, Y_i), $i = 1, \ldots, n$, then the posterior distribution of γ is given by (proportionality constant omitted)

$$f(\gamma|\mathbf{Y}) \propto \prod_{i=1}^{n} \left(\frac{e^{(x_i-\mu)/\sigma}}{1+e^{(x_i-\mu)/\sigma}} \right)^{Y_i} \left(\frac{1}{1+e^{(x_i-\mu)/\sigma}} \right)^{1-Y_i} e^{((\mu-\mu_0)^2)/-2\tau^2} e^{-\sigma/\theta_0}.$$

The maximum-a-posteriori (MAP) estimate of γ is given by

$$\hat{\gamma}^{(n)} = (\hat{\mu}^{(n)}, \hat{\sigma}^{(n)}) = \arg\max_{\gamma} \log f(\gamma|\mathbf{Y}).$$

Thus, the proposed sequential design to estimate the pth quantile becomes $x_{n+1} = \hat{\mu}^{(n)} + \hat{\sigma}^{(n)} \log(p/(1-p))$. Because MAP estimates of γ are used, we call this sequential design procedure as Wu-MAP.

2. Simulation results

The performance of Wu-MAP is evaluated and compared with the other methods using the same simulation settings as reported by Wu and Tian. Two distributions are used to generate the data: the normal distribution $\Phi((x-\mu)/\sigma)$ and the logistic distribution $LG((x-\mu)/(\sigma/1.8138))$ with $\mu = 10$ and $\sigma = 1$, where $LG(z) = (1+e^{-z})^{-1}$. In the work of Wu and Tian, the "guessed" values of μ and σ are denoted by μ_g and σ_g. Thus, we obtain $\mu_0 = \mu_g$ and $\theta_0 = \sigma_g$. Moreover, the simulation settings of 3pod uses $\mu_{\max} = \mu_g + 4\sigma_g$ and $\mu_{\min} = \mu_g - 4\sigma_g$. Equating this range to 8τ, we obtain $\tau = \sigma_g$.

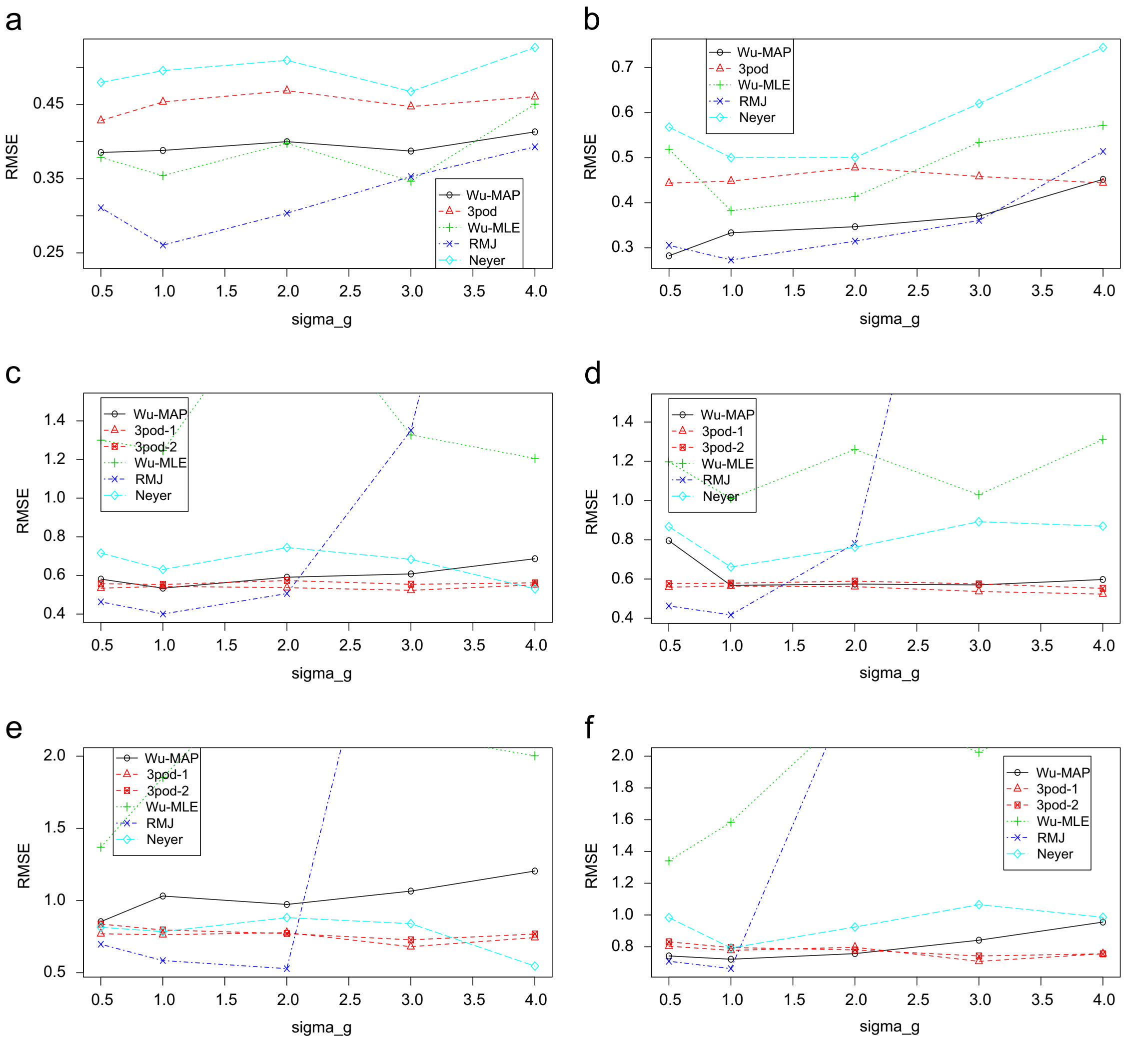

Fig. 1. RMSE for normal distribution. (a) Estimate $x_{0.9}$ with $n = 40$, $\mu_g = 9$, (b) estimate $x_{0.9}$ with $n = 40$, $\mu_g = 11$, (c) estimate $x_{0.99}$ with $n = 60$, $\mu_g = 9$, (d) estimate $x_{0.99}$ with $n = 60$, $\mu_g = 11$, (e) estimate $x_{0.999}$ with $n = 80$, $\mu_g = 9$ and (f) estimate $x_{0.999}$ with $n = 80$, $\mu_g = 11$.