Available online at www.sciencedirect.com
ScienceDirect
Journal of Statistical Planning and Inference 138 (2008) 2607–2614
journal of statistical planning and inference
www.elsevier.com/locate/jspi

Extending the volatility concept to point processes

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Available online 12 March 2008

Abstract

Volatility is a vague concept that can be made precise in a number of ways. This paper investigates the concept for point processes in the time domain. There is review of the time series case, then a volatility measure is developed. Next it is applied to some biomedical data based, running rate intervals of the heart beating. It is seen in the work that in some cases time series volatility might be generated by point process volatility.

MSC: primary 62M10; 60G55; secondary 62M99; 91B30

Keywords: Atrial fibrillation; Point process; Running rate intervals; Time series

"So I think that in time series you can't write in a definitive fashion. The field is still going to have a lot of changes in it."

T. W. Anderson quoted in DeGroot (1990).

1. Introduction

Ted Anderson has been a statistical beacon throughout so many of our academic careers. That light now includes the sparkling talk that he gave recently at age 90. This paper concerns a concept from economic time series analysis, one of Ted's many specialties.

Volatility is an important topic for time series analysis generally and for financial series work particularly. It is a vague concept, capable of being formalized in a variety of ways. One persistent notion is local in time variability, that is, things are changing a lot as time advances. Volatility has been measured by the local standard deviation, $\sigma(t)$, or the local variance, $\sigma(t)^2$ for series such as the returns of a security or a market index. Commonly, the higher the volatility the riskier the security, and thus volatility is important information for a security owner or prospective buyer.

The study of volatility can lead to better forecasting of a series, to better understanding of the past, and to better assessment of risk. For example in insurance considerations the safety loaded pure risk premium can take the form

$$\lambda_1 p(t) + \lambda_2 \sigma(t) + \lambda_3 \sigma(t)^2,$$

where the λ's are weights, $p(t)$ is the fair premium, and $\sigma(t)$ and $\sigma(t)^2$, are volatility measures at time t. One reference to the insurance case is Daykin et al. (1994).

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doi:10.1016/j.jspi.2008.03.003

Another measure of financial risk is the Value at Risk (VaR). It has been defined as the maximum expected loss over a specified time period with a given confidence level of occurring. Losses in the time horizon are only to exceed the VaR with prespecified probability α, see Tsay (2002) or Bouchard and Potters (2003). Estimates of a measure of volatility such as $\sigma(t)$ are required for VaR estimation.

The motivating consideration of this present work is the study of the concept of volatility in the point process case. By a point process is meant a non-decreasing sequence of time points, $\{\tau_j\}$, $j = 0, \pm 1, \pm 2, \ldots$. It seems worth studying an extension to point processes for reasons including:

(a) It appears that generally in the study of time series something more can be learned about that case by studying point process analogs.
(b) Point processes are the building blocks of many other processes. This has the implication that volatility in a time series may be generated from volatility in a latent point process. (This will be illustrated later in the paper.)
(c) Point processes are an interesting data type in their own right.
(d) The subject matters of finance and time series can be brought into the point process case by imagining that there is a common cost associated with each event.
(e) One might chose to replace the point process by a 0–1 valued discrete time series.

Concepts desired in a study of volatility might include: description, detection, formalization, prediction. In this work two measures are proposed, one a function of time and the other a function of a point's index number.

The investigation provided is partly empirical, in that there is study of a biomedical data set. In fact there are many economic and financial empirical examples of volatility analysis of time series, see Bouchard and Potters (2003), and Tsay (2002) for example.

The layout of the paper includes: discussion of the time series concept, then the point process case. There is also material on risk analysis and general discussion.

2. The time series case

2.1. Time series background

The work begins with reference to the development of the volatility concept in the financial world.

On their website Merrill Lynch provides the following definition,

> "*Volatility*. A measure of the fluctuation in the market price of the underlying security. Mathematically, volatility is the annualized standard deviation of returns."

The definition refers to "returns". For a financial entity they are defined as follows: for $t = 0, \pm 1, \pm 2, \ldots$, if P_t denotes the price of the security of interest at time t, the return is

$$Y_t = (P_t - P_{t-1})/P_{t-1}. \tag{1}$$

This definition fits in with the usage of volatility over many years. Return is also sometimes taken to be

$$\log P_t/P_{t-1}.$$

The two definitions are close if the values P_t are not changing quickly.

An empirical formula for volatility at time t is provided by

$$\text{se}\{Y_s \mid s \text{ near } t\}, \tag{2}$$

or its square, with se denoting standard error.

In the time series case there are model-based formulas as well. Consider the GARCH(P, Q) series Y_t given by

$$Y_t = \mu_t + \sigma_t \varepsilon_t, \tag{3}$$